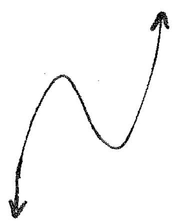
§ 5.5. Absolute Extrema

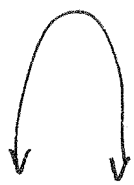
Def'n. If $f(c) \geq f(x)$ for all x in D_f, then $f(c)$ is called the absolute maximum of f.

If $f(c) \leq f(x)$ for all x in D_f, then $f(c)$ is called the absolute minimum of f.

Exs.

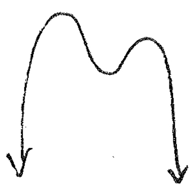


neither

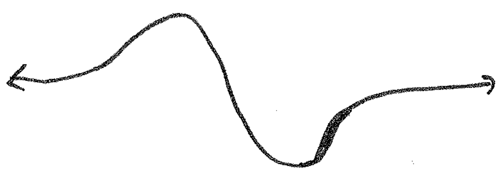


max only

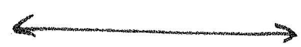
min only



max only



max and min



max = min

Thm. Extreme Value Thm

A function f that is continuous on a closed interval $[a,b]$ has both an absolute max and min on $[a,b]$.

Thm. If an absolute extreme value exists, it must occur at a critical value or at endpoints.

Ex. Find the abs. max/min of

$$f(x) = x^3 + 3x^2 - 9x - 7$$

on $[-6,4]$,

$[-4,2]$,

$[-2,2]$.

Ex. Second derivative test.

~~If f' switches signs and f'' is~~

~~Also~~

If $f'(c) = 0$ and $f''(c) > 0$, then $f(c)$ is a local min.

If $f'(c) = 0$ and $f''(c) < 0$, then $f(c)$ is a local max.

If $f'(c) = 0$ and $f''(c) = 0$, then test does not apply.

Use SDT to find local min/max of $f(x) = e^x - 5x$.

Ex. Use SDT to help find max/min:

61. $f(x) = x^4 - 4x^3 + 5$ on $[0,4]$

On an open interval: May not have both.

Ex. abs. max. on $(0,\infty)$ $f(x) = 5x - 2x\ln x$

Ex. abs. min. on $(0,\infty)$ $f(x) = e^x/x^2$

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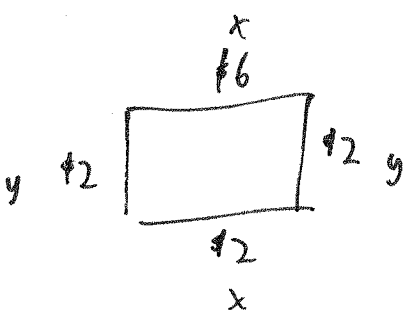
§ 5.6. Optimization

Ex. Area and Perimeter:

Home owner has \$320 to spend on fence for rectangle shaped garden. Three sides are wire \$2/ft.
The fourth side is pickett \$6/ft.
Find the dimensions that would maximize area.



$$6x + 2x + 2y + 2y = 8x + 4y = \$320$$

$$C = 8x + 4y$$

$A = xy$ w/ $4y = 320 - 8x$

$y = 80 - 2x$

$$A = x(80 - 2x)$$
$$= 80x - 2x^2$$

$$\frac{dA}{dx} = 80 - 4x = 0$$
$$80 = 4x$$
$$x = 20$$

then $y = 40$

so fenced area should be $\boxed{20 \times 40}$

Ex 4. Find two numbers whose diff is 21 and whose product is a min.

$P = xy \qquad x - y = 21$

$P = x(x-21) = x^2 - 21x$

$P' = 2x - 21 = 0$

$2x = 21$

$x = \frac{21}{2} = 10.5 \qquad y = 31.5$

Ex 8. Find dim. of a rect. w/ area 108, and min. perimeter.

$A = xy \qquad P = 2x + 2y = 2x + \frac{216}{x}$

$108 = xy$

$y = \frac{108}{x} \qquad P' = 2 - \frac{216}{x^2} = 0$

$x^2 = 108$

$x = \sqrt{108}$

$y = \frac{108}{\sqrt{108}} = \sqrt{108}$

Ex 18. RSC sells 1600 cups of coffee/day for \$2.40. study shows every \$0.05 reduction in prize results in 50 more cups sold. Find the optimal price. to max. rev.

~~$p(x) = 2.40 - 0.05x$~~ ~~$s(x) = 1600 + 50x$~~

$p(x) = \$2.40 - 0.05x$

$R(x) = (1600 + 50x)(p(x))$

$= (1600 + 50x)(2.40 - 0.05x)$

$= 1280 - \frac{5}{2}x^2 + 120x - 80x$

$= -\frac{5}{2}x^2 + 40x + 1280$

so $R'(x) = -5x + 40 = 0$

$x = +8$

This means optimal price is $\$2.40 - 8(0.05)$

$= \boxed{\$2.00 \text{ /cup.}}$

End of material for Exam #2!

(Mon 7/16/12)