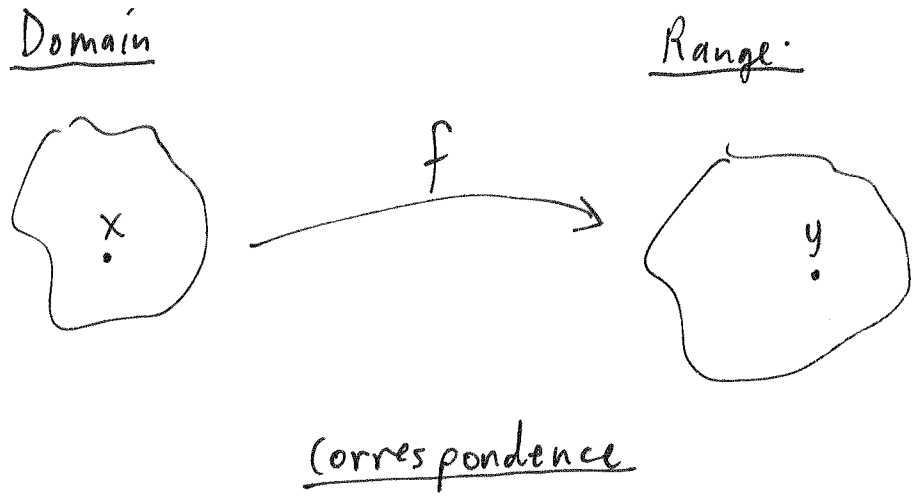


— 4 Jun '12

ch. 2.: Review of key alg. concepts:

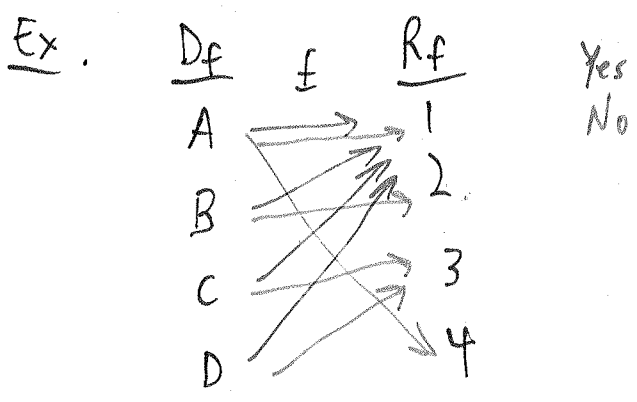
1. Functions

A function is a correspondence between two sets such that every member of the first set corresponds to exactly one member of the second.



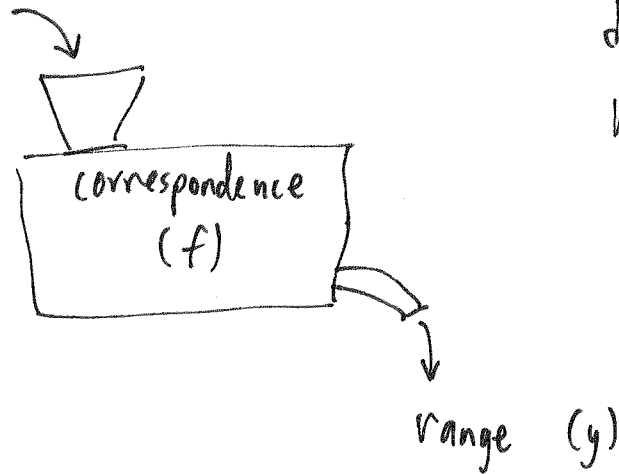
We write $y = f(x)$.

y is the ^{unique} member of the range that corresponds to x (from the domain).



1.1 Functions as machines:

domain
(x)

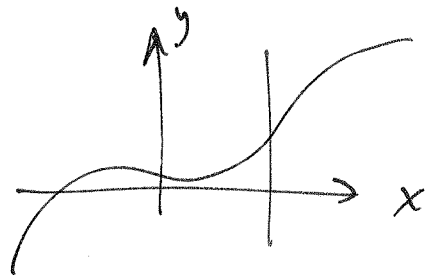


domain: inputs (x)

range: outputs (y)

1.2. Graphs of functions:

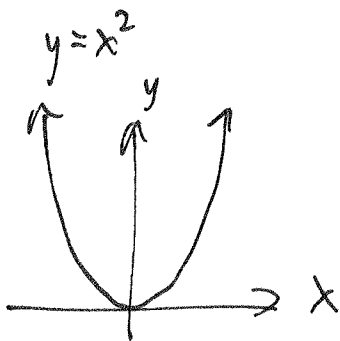
$$y = f(x)$$



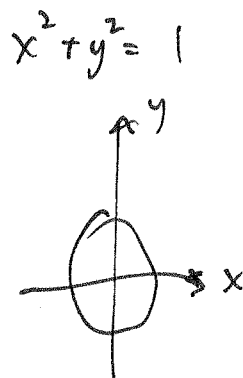
1.3 Vertical Line Test:

If any vertical line crosses a graph more than once, then the graph is not the graph of a function.

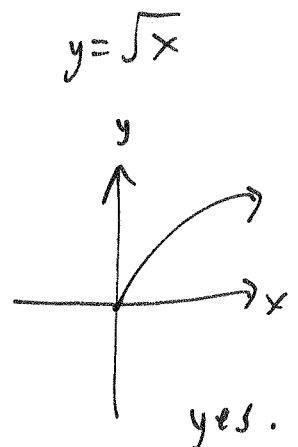
Exs.



Yes.



No.



yes.

$y^2 = x$. No.

$\sqrt{2}$

1.4. Domain: Find the domains of the functions.

Ex. $y = \sqrt{x-2}$

$$f(x) = \frac{x+3}{x^2-7x+12}$$

Major restrictions: $\sqrt{\quad}$, and \div
but also logs. (+bs)

1.5. Evaluating functions:

Ex. $f(x) = x^2 + 2x + 1$

$$f(5) = 5^2 + 2(5) + 1$$

$$= 25 + 10 + 1$$

$$= 36$$

$$f(a) = a^2 + 2a + 1$$

$$f(x+h) = (x+h)^2 + 2(x+h) + 1$$

$$= (x+h)(x+h) + 2x + 2h + 1$$

$$= x^2 + 2xh + h^2 + 2x + 2h + 1$$

Ex. $f(x) = x^5$

$$f(2) = 2^5 = 32$$

$$f(x+h) = \text{PASCAL'S TRIANGLE!}$$

$$\begin{array}{ccccccc} & & & & 1 & & & & \\ & & & & & 1 & & & \\ & & & & & & 1 & & \\ & & & & & & & 1 & \\ & & & & & & & & 1 \\ & & & & & & & & & 1 \\ & & & & & & & & & & 1 \\ & & & & & & & & & & & 1 \\ & & & & & & & & & & & & 1 \\ & & & & & & & & & & & & & 1 \\ & & & & & & & & & & & & & & 1 \end{array}$$

$$= x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$$

1.6. Difference Quotients!

The essence of differential calculus (as we will soon see).

For any function $y=f(x)$, its difference quotient is

$$DQ(f) = \frac{f(x+h) - f(x)}{h} \quad \text{for } h \neq 0.$$

Ex. $f(x) = 3x + 5$

$$\begin{aligned} f(x+h) &= 3(x+h) + 5 \\ &= 3x + 3h + 5 \end{aligned}$$

$$\begin{aligned} f(x+h) - f(x) &= 3x + 3h + 5 - (3x + 5) \\ &= 3x + 3h + 5 - 3x - 5 \\ &= 3h \end{aligned}$$

$$DQ(f) = \frac{f(x+h) - f(x)}{h} = \frac{3h}{h} = \boxed{3}$$

Ex. $f(x) = x^2 + 2x + 1$

$$\begin{aligned} f(x+h) &= (x+h)^2 + 2(x+h) + 1 \\ &= x^2 + 2xh + h^2 + 2x + 2h + 1 \end{aligned}$$

$$\begin{aligned} f(x+h) - f(x) &= x^2 + 2xh + h^2 + 2x + 2h + 1 - (x^2 + 2x + 1) \\ &= 2xh + h^2 + 2h \end{aligned}$$

$$DQ(f) = \frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 + 2h}{h} = \frac{h(2x + h + 2)}{h} = \boxed{2x + 2 + h}$$

1.7. Transformations of graphs:

Start with a known graph, called a parent function (or basic function) $y = p(x)$.

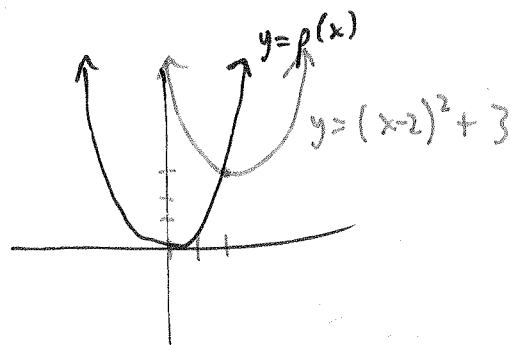
Horizontal Translations let $c > 0$

1. $y = f(x) = p(x+c)$: shift left c .
2. $f(x) = p(x-c)$: right c .
3. $f(x) = p(x) + c$: up c
4. $f(x) = p(x) - c$: down c .

Ex. $y = (x-2)^2 + 3$

$p(x) = x^2$

right 2, up 3.



Ex. $y = \sqrt{x+3} + 1$

Ex. $y = -|x| - 2$

←!

$f(x) = -p(x)$

reflect over x-

$f(x) = p(-x)$

reflect over y-

2. Completing the Square

Most powerful tool for working with quadratic functions (parabolas).

2.1. Vertex of Parabola

Ex. $f(x) = x^2 + 4x + 9$

$$\left. \begin{array}{l} b=4 \\ \frac{b}{2}=2 \\ \left(\frac{b}{2}\right)^2=4 \end{array} \right\} \begin{array}{l} f(x) = (x^2 + 4x + 4) - 4 + 9 \\ = (x+2)^2 + 5 \\ \quad \uparrow \quad \uparrow \\ \text{vertex of parabola!} \end{array}$$

$$\boxed{V = (-2, 5)}$$

Standard equation of parabola: $f(x) = a(x-h)^2 + k$
(h,k) = vertex.

Ex. $f(x) = x^2 - 5x + \frac{1}{2}$

$$\left. \begin{array}{l} b=-5 \\ \left(\frac{b}{2}\right) = -\frac{5}{2} \\ \left(\frac{b}{2}\right)^2 = \frac{25}{4} \end{array} \right\} \begin{array}{l} f(x) = \left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} + \frac{1}{2} \\ = \left(x - \frac{5}{2}\right)^2 - \frac{23}{4} \\ \quad \uparrow \quad \uparrow \end{array}$$

$$\boxed{V = \left(\frac{5}{2}, -\frac{23}{4}\right)}$$

Ex. $f(x) = -2x^2 + 16x - 4 = -2(x^2 - 8x + 2)$

$$\left. \begin{array}{l} b=-8 \\ \frac{b}{2}=-4 \\ \left(\frac{b}{2}\right)^2=16 \end{array} \right\} \begin{array}{l} f(x) = -2[(x^2 - 8x + 16) - 16 + 2] \\ = -2[(x-4)^2 - 14] \\ = -2(x-4)^2 + 28 \end{array}$$

$$\boxed{V = (4, 28)}$$

2.2. Solving eqns using Cts.

Ex. Solve: $x^2 + 4x - 1 = 0$

$$(x+2)^2 - 5 = 0$$

$$(x+2)^2 = 5$$

$$x+2 = \pm\sqrt{5}$$

$$\boxed{x = -2 \pm \sqrt{5}}$$

Ex. Solve: $9x^2 - 18x + 81 = 0$

$$9(x^2 - 2x + 9) = 0$$

$$(x-1)^2 - 10 = 0$$

$$(x-1)^2 = 10$$

$$x-1 = \pm\sqrt{10}$$

$$\boxed{x = +1 \pm \sqrt{10}}$$

3. Logs and Exponentials

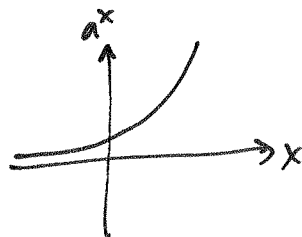
3.1. Exponential Equations (functions)

$$f(x) = a^x \quad \text{for } a > 0, a \neq 1.$$

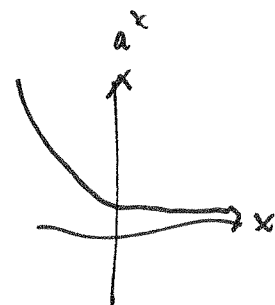
the variable is in the exponent.

domain: $(-\infty, \infty)$

range: $(0, \infty)$



for $a > 1$



for $a < 1$.

3.2. Exponential Equations

Ex. $2^x = 32$ solve for x .

$$2^x = 2^5$$

$$\boxed{x=5}$$

Ex. solve for x : $9^{x+2} = 3^x$

$$(3^2)^{x+2} = 3^x$$

$$3^{2x+4} = 3^x$$

$$2x+4 = x$$

$$\boxed{x=-4}$$

3.3. Logarithms

Definition. $\log_a x = y$ means $a^y = x$

$$a \neq 1, a > 0.$$

Ex. $\log_2 8 = ?$

Write $\log_2 8 = y \Rightarrow 2^y = 8$

$$2^y = 2^3$$

$$\boxed{y=3}$$

so $\log_2 8 = 3.$

Logs are exponents.

$$\log_{10} = \log = \text{common log.}$$

$$\begin{cases} \log_a 1 = 0 \\ \log_a a = 1 \end{cases}$$