

Instructions: Complete all problems, showing all relevant work. Reduce or simplify answers as much as possible without using a calculator. Circle or clearly mark your final answers.

1. [5 points each] Find the derivative of each function. Simplify if necessary.

a. $f(x) = 4x^{30} + 2x^9 - 8x^2 + 15$

$$f'(x) = 120x^{29} + 18x^8 - 16x$$

b. $y = e^{x^3+2x^2+x+100}$

$$y' = e^{x^3+2x^2+x+100} (3x^2 + 4x + 1)$$

c. $g(x) = \frac{\ln x}{x^2}$

$$g'(x) = \frac{x^2(\frac{1}{x}) - \ln x(2x)}{(x^2)^2} = \frac{x - 2x \ln x}{x^4} = \frac{x(1 - 2 \ln x)}{x^4} = \boxed{\frac{1 - 2 \ln(x)}{x^3}}$$

d. $h(x) = x^3 e^x$

$$h'(x) = x^3 e^x + 3x^2 e^x = x^2 e^x (x+3)$$

e. $y = \ln(2x^2 + 4x)$

$$y' = \frac{4x+4}{2x^2+4x} = \boxed{\frac{2x+2}{x^2+2x}}$$

1 (cont'd). Find the derivative of each function. Simplify if necessary.

f. $f(x) = \log_{17}(7x)$

$$f'(x) = \frac{7}{7x \ln(17)} = \boxed{\frac{1}{x \ln(17)}}$$

g. $y = \sqrt{x^2 - 9}$

$$y' = \frac{2x}{2\sqrt{x^2 - 9}} = \boxed{\frac{x}{\sqrt{x^2 - 9}}}$$

h. $f(x) = \frac{x^2 + x + 1}{x^3} = x^{-1} + x^{-2} + x^{-3}$

$$f'(x) = -x^{-2} - 2x^{-3} - 3x^{-4}$$

2. [10 points] Find an equation for the tangent line to the graph of $x^2 + y^2 = 25$ at the point (3, 4).

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{y} \quad \text{at } (3, 4) = \frac{-3}{4} = m$$

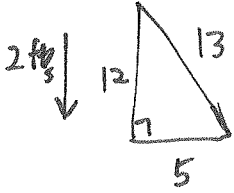
$$(y - y_1) = m(x - x_1)$$

$$y - 4 = \frac{-3}{4}(x - 3)$$

$$y = \frac{-3}{4}x + \frac{9}{4} + 4 \quad \text{so}$$

$$\boxed{y = \frac{-3}{4}x + \frac{25}{4}}$$

3. [10 points] A 13 foot long ladder is placed against a wall. If the top of the ladder is sliding down the wall at a rate of 2 ft/s, how fast is the bottom of the ladder sliding away from the wall when the bottom is 5 feet from the base of the wall?



$$x^2 + y^2 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$\left. \begin{array}{l} z=13 \\ x=5 \end{array} \right\} y=12$$

$$\frac{dy}{dt} = -2$$

$$\frac{dz}{dt} = 0$$

$$\frac{dx}{dt} = ?$$

$$5 \frac{dx}{dt} + 12(-2) = 0$$

$$\frac{dx}{dt} = \frac{24}{5} = \boxed{4.8 \text{ ft/s}}$$

4. [5 points each] Evaluate the limits. Show your work.

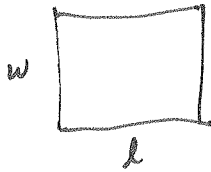
a. $\lim_{x \rightarrow \infty} \frac{\ln x}{x^3} = \frac{\infty}{\infty} \quad L'H$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{3x^2} = \lim_{x \rightarrow \infty} \frac{1}{3x^3} = \frac{1}{\infty} = \boxed{0}$$

b. $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = \frac{e^0 - 1}{0} = \frac{1 - 1}{0} = \frac{0}{0} \quad L'H$

$$= \lim_{h \rightarrow 0} \frac{e^h}{1} = \frac{e^0}{1} = \frac{1}{1} = \boxed{1}$$

5. [10 points] Find the dimensions of a rectangle with an area of 216 ft^2 that has the minimum perimeter. [Area = $l \cdot w$, Perimeter = $2l + 2w$]



$$A = lw = 216 \Rightarrow l = \frac{216}{w} \text{ or } w = \frac{216}{l}$$

$$P = 2l + 2w = 2l + 2\left(\frac{216}{l}\right)$$

$$P' = 2 - \frac{432}{l^2} = 0$$

$$l^2 = \frac{432}{2} = 216$$

$$l = \sqrt{216} = 6\sqrt{6}$$

$$w = \sqrt{216} = 6\sqrt{6}$$

6. [20 points] Let $f(x) = \frac{3x}{x+2}$. Find all of the following.

- a. What is the domain of f ?

$$D_f : \{x \mid x \neq -2\}$$

- b. Find $f'(x)$. Find all critical values of f .

$$f'(x) = \frac{(x+2)(3) - (3x)}{(x+2)^2} = \frac{6}{(x+2)^2}$$

CVs: top: none bot: $x=2$

- c. Determine the intervals on which f is increasing and/or decreasing.



$$f' > 0 \text{ always.}$$

$$\text{inc: } (-\infty, -2) \cup (-2, \infty)$$

$$\text{dec: never}$$

- d. Find all local max and min of f , if any exist.

None

Recall: $f(x) = \frac{3x}{x+2}$

e. Find $f''(x)$. Find all critical values of f' .

$$f''(x) = (6(x+2)^{-2})' = -12(x+2)^{-3}(1) = \boxed{\frac{-12}{(x+2)^3}}$$

CVs: top: none
 bott: $x = -2$

f. Determine the intervals on which f is concave up and/or concave down.



$$f''(-3) = \frac{-}{-} = +$$

$$f''(0) = \frac{-}{+} = -$$

Conc up: $(-\infty, -2)$
 Conc down: $(-2, \infty)$

g. Find all inflection points of f , if any exist.

None b/c -2 is not in D_f !

h. Find all horizontal and vertical asymptotes of f , if any exist.

VA: $x = -2$

HA: $\lim_{x \rightarrow \pm\infty} \frac{3x}{x+2} = 3$ so $y = 3$ is HA.

i. Sketch the graph of f .

Need some pts:

$$f(0) = \frac{0}{2} = 0 \Rightarrow (0, 0)$$

$$f(-4) = \frac{-12}{-2} = 6$$

