

Instructions: Complete all problems, showing all relevant work. Reduce or simplify answers as much as possible without using a calculator. Circle or clearly mark your final answers.

1. [12 points] Let $f(x) = x^2 + 6x + 1$.

a) Write f in *vertex form* by completing the square.

$$\begin{array}{l}
 b=6 \\
 \frac{b}{2}=3 \\
 \left(\frac{b}{2}\right)^2=9
 \end{array}
 \left.
 \begin{array}{l}
 \\
 \\
 \end{array}
 \right\}
 \begin{array}{l}
 f(x) = (x^2 + 6x + 9) - 9 + 1 \\
 \boxed{f(x) = (x+3)^2 - 8}
 \end{array}$$

b) What is the vertex of the parabola given by $y = f(x)$?

$$\boxed{V = (-3, -8)}$$

2. [12 points] Solve each of the equations for x :

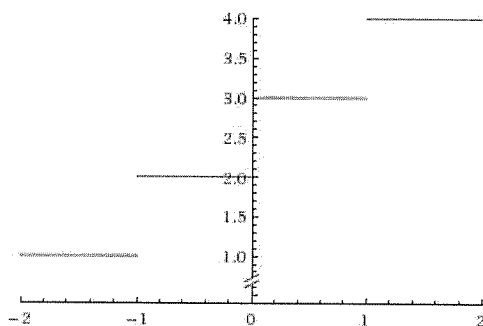
a) $81 = 9^{x^2-x}$

$$\begin{array}{l}
 \cancel{9}^2 = \cancel{9}^{x^2-x} \\
 2 = x^2 - x \\
 x^2 - x - 2 = 0 \\
 (x-2)(x+1) = 0 \\
 \boxed{x=2, x=-1}
 \end{array}$$

b) $\log_2(x-2) = 5$

$$\begin{array}{l}
 x-2 = 2^5 \\
 x-2 = 32 \\
 \boxed{x=34}
 \end{array}$$

3. [12 points] Calculate the limits using the graph of $y = f(x)$.



a) $\lim_{x \rightarrow -1^-} f(x) = 1$

c) $\lim_{x \rightarrow 0} f(x) = \text{DNE}$

b) $\lim_{x \rightarrow -1^+} f(x) = 2$

d) $\lim_{x \rightarrow \frac{1}{2}} f(x) = 3$

4. [12 points] Determine where the function is continuous. Determine the type of each of its discontinuities (hole, asymptote, jump).

$$f(x) = \frac{x^2 - 5x + 6}{x^2 - x - 2} = \frac{(x-3)(x-2)}{(x-2)(x+1)}$$

$$x^2 - x - 2 \neq 0$$

$$(x-2)(x+1) \neq 0$$

Domain: $x \neq 2, x \neq -1$

Discontinuities:

$x = 2$: hole
 $x = -1$: asymptote

$$f(x) = \frac{x^2 + 6x + 9}{x^2 - 9} = \frac{(x+3)(x+3)}{(x+3)(x-3)}$$

5. [12 points] Let $f(x) = \frac{x^2 + 6x + 9}{x^2 - 9} = \frac{(x+3)(x+3)}{(x+3)(x-3)} = \frac{(x+3)(x-3)}{(x-3)(x-3)}$

a) Find the domain of f .

$$x^2 - 9 \neq 0$$

$$(x+3)(x-3) \neq 0$$

$$\boxed{x \neq \pm 3}$$

b) Find $\lim_{x \rightarrow -3} f(x)$.

$$= \lim_{x \rightarrow -3} \frac{x+3}{x-3} = \frac{0}{-6} = \boxed{0}$$

c) Find $f(-3)$. Is f continuous at $x = -3$?

$$f(-3) = \frac{0}{0} = \underline{\text{undef.}} \quad \underline{\text{Not cont.}}$$

d) Find $\lim_{x \rightarrow \infty} f(x)$.

$$\lim_{x \rightarrow \infty} \frac{x^2 + 6x + 9}{x^2 - 9} = \frac{1}{1} = \boxed{1}$$

6. [12 points] Let $f(x) = \begin{cases} x^2, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$

Use the limit definition of continuous to determine if $f(x)$ is continuous at the point $x = 0$.

$$\left. \begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} x^2 = 0^2 = 0 \\ \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} \sqrt{x} = \sqrt{0} = 0 \end{aligned} \right\} \lim_{x \rightarrow 0} f(x) = 0$$

$$f(0) = \sqrt{0} = 0$$

so $f(0) = \lim_{x \rightarrow 0} f(x) = 0$ and f is
cont. at $x = 0$.

7 - 8. Let $f(x) = x^3 + x$

7. [12 points] Find the *difference quotient* for the function $f(x)$.

$$f(x+h) = (x+h)^3 + (x+h) = x^3 + 3x^2h + 3xh^2 + h^3 + x + h$$

$$f(x+h) - f(x) = x^3 + 3x^2h + 3xh^2 + h^3 + x + h - x^3 - x$$

$$= 3x^2h + 3xh^2 + h^3 + h$$

$$= h(3x^2 + 3xh + h^2 + 1)$$

$$\frac{f(x+h) - f(x)}{h} = \frac{h(3x^2 + 3xh + h^2 + 1)}{h} = \boxed{3x^2 + 3xh + h^2 + 1}$$

8. [12 points] Use your answer to problem 7 and the *limit definition of derivative* to find the following:

$$a) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2 + 1) = \boxed{3x^2 + 1}$$

$$b) f'(3) = 3(3)^2 + 1 = 27 + 1 = \boxed{28}$$

c) An equation for the tangent line to $y = f(x)$ at $x = 3$.

$$y - y_1 = m(x - x_1)$$

$$y - 30 = 28(x - 3)$$

$$y = 28x - 84 + 30$$

$$\boxed{y = 28x - 54}$$

$$f(3) = 3^3 + 3 = 27 + 3 = 30$$