

MATH555  
Differential Equations  
Practice Exam I

1. Find the value of  $y_0$  for which the solution of the initial value problem

$$y' - y = 1 + 2 \cos t, \quad y(0) = y_0$$

remains finite as  $t \rightarrow \infty$ .

2. Find the solution of the given initial value problems in explicit form, and determine the interval in which the solution is defined and *correct*.

$$x^2 y' + \sqrt{1 - y^2} = 0, \quad y(1) = 0.$$

3. Solve the given initial value problem.

$$\frac{dy}{dx} = \frac{y^3}{x^3 + xy^2}, \quad y(1) = 1.$$

4. Find an integrating factor and solve the given initial value problem.

$$2xydx + (3x^2 + 4y)dy = 0, \quad y(0) = 1.$$

5. A raindrop falling in the air is subject to gravity (downward) and drag (upward). Assume that the drag is proportional to the speed of the raindrop. The downward acceleration is given by

$$a = g - kv,$$

where  $g$  and  $k$  are constants. The raindrop starts from still.

(a) Find the terminal (equilibrium) velocity  $v_T$  of the rain drop, and the time for the velocity of the raindrop to reach half of  $v_T$ .

(b) Denote by  $x$  the distance travelled by the raindrop. Use the relation  $dv/dt = v(dv/dx)$  to write the equation of motion in terms of  $v$  and  $x$ . Find the distance travelled by the raindrop by the time its velocity reaches half of  $v_T$ .

6. Solve the given initial value problem.

$$y' = \frac{2y}{x} + y^2 + x^2 y. \quad y(1) = 1.$$