Supplement to ~~Lecture~~ Lecture – G.E. with partial pivoting (p.p.)

Math 551 – DeLillo, 2/9/12

PA = LU

$$A = \begin{bmatrix} 3 & 17 & 10 \\ 2 & 4 & -2 \\ 6 & 18 & -12 \end{bmatrix}$$

entry of largest magnitude in col. 1 $\rightarrow 6$

$$P_1 A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A = \begin{bmatrix} 6 & 18 & -12 \\ 2 & 4 & -2 \\ 3 & 17 & 10 \end{bmatrix}$$

p.p.: exchange 1st + 3rd row

$$L_1 P_1 A = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix} P_1 A = \begin{bmatrix} 6 & 18 & -12 \\ 0 & -2 & 2 \\ 0 & 8 & 16 \end{bmatrix}$$ p.p. exchange

Note (*)
P_1 and P_2 can be stored as
$$p = \begin{bmatrix} p(1) \\ p(2) \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \begin{matrix} \leftarrow \text{swap rows } 1+3 \\ \leftarrow \text{swap rows } 2+3 \end{matrix}$$

$$P_2 L_1 P_1 A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} L_1 P_1 A = \begin{bmatrix} 6 & 18 & -12 \\ 0 & 8 & 16 \\ 0 & -2 & 2 \end{bmatrix}$$

$$L_2 P_2 L_1 P_1 A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{4} & 1 \end{pmatrix} \begin{pmatrix} 6 & 18 & -12 \\ 0 & 8 & 16 \\ 0 & -2 & 2 \end{pmatrix} = \begin{bmatrix} 6 & 18 & -12 \\ 0 & 8 & 16 \\ 0 & 0 & 6 \end{bmatrix} =: U$$

$$\therefore\ U = L_2 P_2 L_1 P_1 A = L_2 P_2 L_1 (P_2^{-1} P_2) P_1 A$$

(Note $P_2^{-1} = P_2^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$)

$$= L_2 (P_2 L_1 P_2^{-1}) P_2 P_1 A = L_2 \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right) \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right) A$$

$$= L_2 \left(\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 0 & 1 \\ -\frac{1}{3} & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right) \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{= P_2 P_1 =: P} A$$

3rd stroke of luck:
$$L_2 P_2 L_1 P_1 = L_2 (P_2 L_1 P_2^{-1}) P_2 P_1 = L_2 L_1' P$$

$$= \underbrace{L_2}_{= L_2'} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{3} & 0 & 1 \end{bmatrix}}_{=: L_1' = P_2 L_1 P_2^{-1}} PA$$

$$= L_2' L_1' PA \Rightarrow PA = LU \text{ where } L = L_1'^{-1} L_2'^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{3} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{4} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{3} & -\frac{1}{4} & 1 \end{bmatrix}$$

$$\therefore\ PA = \begin{bmatrix} 6 & 18 & -12 \\ 3 & 17 & 10 \\ 2 & 4 & -2 \end{bmatrix} = LU = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{3} & -\frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 6 & 18 & -12 \\ 0 & 8 & 16 \\ 0 & 0 & 6 \end{bmatrix}$$

$$PA = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 17 & 10 \\ 2 & 4 & -2 \\ 6 & 18 & -12 \end{bmatrix}$$

HW 4 due Th 2/16/12
Find $PA = LU$ using partial pivoting for $A = \begin{bmatrix} 2 & 5 & 5 \\ 6 & 12 & 6 \\ 3 & 8 & 7 \end{bmatrix}$ by a hand calculation, as above.