1. Suppose that $X_1, X_2, \ldots, X_n, \ldots$ are independent and identically distributed random variables with $EX_k = \mu$ and $E(X_k - \mu)^2 = \sigma^2$, $k = 1, 2, \ldots$. Show that, for any positive constant $\varepsilon$,

$$\lim_{n \to \infty} P \left( \left| \frac{\sum_{k=1}^{n} X_k}{n} - \mu \right| \geq \varepsilon \right) = 0.$$

2. The incomplete gamma function, $IG(x, \lambda)$, is defined by

$$IG(x, \lambda) = \int_0^x u^{\lambda-1} \exp(-u) du, \quad x \geq 0,$$

where $\lambda$ is a positive constant. In particular, when $x \to \infty$, $IG(x, \lambda)$ tends to the gamma function $\Gamma(\lambda)$.

(1) Use the Monte Carlo method to write a R program for approximating $IG(x, \lambda)$.

(2) Use your program to find the numerical value of $P(1 \leq X \leq 2)$, where the random variable $X$ follows a gamma distribution with density

$$f(x) = \begin{cases} \frac{1}{\Gamma(3.8)} x^{2.8} \exp(-x), & x \geq 0, \\ 0, & x < 0. \end{cases}$$