“Designing a Traffic Circle”
By
David Bosworth
For
MATH 714
Abstract

We all have had the experience of sitting in a traffic jam, or we’ve seen cars bunched up on a road for some no apparent reason. The use of traffic circles, or sometimes called “Roundabouts”, is a common means of controlling traffic in an intersection. Traffic circles come in either a smaller or larger size depending on the size of the intersection and/or the congestion of the traffic. We can use smaller traffic circles to effectively route and control lower levels of vehicular, since we have traffic flows that are only in one direction within the circle. For larger intersections, however, the situation becomes real complicated and the congestion can easily form at a higher traffic levels. In order for us to find a means of effectively directing traffic within a traffic circle, there are numerous simulation programs based on the traffic dynamics described by Bando et al. By dividing the traffic circle into different sections, we are to apply a realistic model of car interactions within the traffic circle in order to determine the most efficient methods for controlling the traffic flow. Also, we can determine that the best control method for a small traffic circle is for the cars that are already within the traffic circle have to yield to the cars that are entering the traffic circle. For a single lane traffic circle, they have found that the minimum average travel time is approximately 11.26 seconds for cars that are entering at a rate of \( \frac{\text{veh}}{3\text{sec}} \) into a single lane traffic circle of a radius of 40 meter.
1 Introduction

Mankind has used traffic circles to regulate both vehicular traffic and non vehicular traffic in a lot of developing cities around the world. To measure their effectiveness, we have to observe in both their efficiency and their ability to manage relatively large volumes of traffic and the reduction of accidents of all vehicles since they drive in the same direction.

1.1 Current Problems with Traffic Circles

Throughout the world traffic circles can vary significantly in design, even though majority of them feature traffic entering from intersecting streets and flowing in a single direction around a center island. For smaller traffic circles, traffic is typically fairly efficient since the traffic density is usually minimal and the congestion is avoided since the cars need not, and quite often are not allowed to, switch lanes often. However, for larger traffic circles, the traffic becomes considerably more congested due to the high traffic volume and the necessity to switch lanes.

1.2 The Goal of the Model

The goal of most traffic circle models is to analyze the effectiveness of different traffic control devices (such as, traffic lights, stop signs, yield signs, and pedestrian signals), on the traffic circles genuinely in terms of efficiency of cars moving through them. We can also use simulations on an expansive set of different traffic circle designs in order to learn the most efficient control for each behavior of the
traffic within a traffic circle which can varies considerably with the parameters of the traffic circle.

First we need to derive definitions of the quantities if we wish to optimize. The following definitions are adopted from [1].

**Definition 1. Flow rate** (in $\text{veh/hr}$) is how many vehicles that go through a specific area in a unit of time.

**Definition 2.** We can define the **Average Travel Time** (ATT) of a traffic system to be the average time it takes a vehicle to exit after having entered the system.

Thus the goal of most models is to determine what use of traffic devices at a particular traffic circle that will have the lowest and highest capacity at a given entrance flow.

2 A Theoretical Model for Traffic Flow

We can consider most traffic flow models as either in the microscopic scale or the macroscopic scale. In terms of macroscopic models we can model traffic by comparing it to fluid dynamics, blood flow in a human body, or particle dynamics. Microscopic models instead deal with individual cars and drivers, and can usually fall into continuous or cellular automata models. When we deal with traffic situations with very sensitive dependency on system disturbances, continuous models have found to be much more useful and more accurate [3].
2.1 The Bando Acceleration for Car-Following Traffic Flow.

A very well known car-following model for traffic patterns was described by Bando et al in [2]. This model is well known for showing complicated congestion patterns and can be relatively simple with several parameters [2] [3], for which certain appropriate values have been established. Let us define $x_i$ to be the position of car $i$. The model predicts that a certain vehicle will after its acceleration to so that an ideal headway, which will depend on the velocity of the car and the position of the car ahead of it, is maintained. The differential equation that describes the linear motion is

$$x_{i}''(t) = k[V(\Delta x_n(t)) - x'(t)],$$

where $\Delta x = x_{n+1} - x_n$ is the headway between cars $i$ and $(i-1)$ and $V(\Delta x_n)$ is the optimal velocity function and is defined as $V(\Delta x_n(t)) = V_1 + V_2 \tanh[C_1(\Delta x_n(t) - l_c) - C_2].$

Helbing et al estimated the values of these parameters as $V_1 = 6.75 \frac{m}{s}$, $V_2 = 7.91 \frac{m}{s}$, $C_1 = 0.13 m^{-1}$, and $C_2 = 1.57$.

When we consider a system with a considerable volume of vehicle traffic, the nonlinear nature of this system combined with the large number of variables makes it largely complicated even though we can find the stability of the steady state solutions.
2.2 The Stability of the Bando Model

We need to ask ourselves whether the Bando model is stable, i.e. whether the solutions for equally spaced cars traveling in a circular track, all with the same velocity, and such that \( x'(t) = V(\Delta x) \) for each car, is a stable solution. If we solve the linearized system, we can find that the steady state solution to be stable if \( V'(b) < \frac{k}{2} \) and unstable if \( V'(b) > \frac{k}{2} \) where \( b \) is the steady state headway distance [2]. When we evaluate the expression for \( V'(b) \) using the stated parameter values, we find that this steady state solution to the model is stable in a circle of radius 40 meter when there are 10 or fewer cars in the circle.

3 Summary

We can use computer simulations to effectively measure the effectiveness of traffic circle control mechanisms. Any traffic engineer may use the discrete traffic flow model to best calculate which control mechanism that will suit for any given traffic scenario. In most basic case, a typical traffic circle consists of one lane (See Figure 1) with four single entry lanes symmetrically placed, entering the circle, we find that cars that are yielding to traffic already in the circle produce the lowest average time of traveling through the traffic circle.
Figure 1: A rendering of the simplest single lane traffic circle.
References

