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An Improved Car-Following Model for Multiphase Vehicular Traffic Flow and Numerical Tests

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Abstract This paper attempts to introduce an improved difference model that modifies a car-following model, which takes the next-nearest-neighbor interaction into account. The improvement of this model over the previous one lies in that it performs more realistically in the dynamical motion for small delay time. The traffic behavior of the improved model is investigated with analytic and numerical methods with the finding that the new consideration could further stabilize traffic flow. And some simulation tests verify that the proposed model can demonstrate some complex physical features observed recently in real traffic such as the existence of three phases: free flow, coexisting flow, and jam flow; spontaneous formation of density waves; sudden flow drop in flow-density plane; traffic hysteresis in transition between the free and the coexisting flow. Furthermore, the improved model also predicts that the stable state to relative density in the coexisting flow is insusceptible to noise.

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Key words: car-following models, density waves, traffic flow

1 Introduction

Over the past decade, many physicists have been interested in the traffic dynamics, especially traffic congestion. To explain this phenomenon, a lot of studies in the mathematical models have been done in describing the dynamics of discrete groups of road vehicles. Among these car-following models describe the dynamical behavior of a line of closely spaced road vehicles traveling in a single lane without overtaking. This makes them particularly suitable mathematical models for studying the motion of congested traffic. Therefore, car-following models are not only of great importance with regard to an autonomous cruise control system, but also have emerged as important evaluation tools for intelligent transportation system (ITS) strategies since the early 1990s.

In this paper, a car-following model with consideration of the next-nearest-neighbor interaction is improved by taking velocity difference into account. Simulation result shows that this model can perform more realistically in a pre-specified acceleration and deceleration pattern for small delay time. Evidence shows that this new model is theoretically an improvement over the generalized optimal velocity model (GOVM) in stability of traffic flow. Moreover, it demonstrates and explains some complex physical phenomena observed recently in real traffic flows.

The remaining parts of the paper are organized as follows. In Sec. 2 we analyze the performance of the previous model in the velocity evolution of a platoon of cars induced by a pre-specified pattern and present the improved car-following model. In Sec. 3 both analytic and numerical methods of this model are used to discuss the stability of traffic flow, and evolution of congestion induced by a small perturbation is carried out to demonstrate three traffic phases. In Sec. 4 numerical simulations are carried out, in particular, the density waves and their stability are examined. In Sec. 5 we apply this model to reproduce two important features, sharp flow drop and traffic hysteresis, from which we shall see that the proposed model works well for our purpose. Finally we conclude the paper in Sec. 6.

2 Discussion of Previous Models and Modification of GOVM

As basic and important components of microscopic approaches, some car-following theories have been given considerable attention in recent years. In 1995, Bando et al. proposed an important car-following model called the “optimal velocity model” (OVM). In the OVM, the acceleration of the n-th vehicle at time t is determined by the difference between the actual velocity \( v_n(t) \) and an optimal velocity \( V(\Delta x_n(t)) \), which depends on the headway \( \Delta x_n(t) \) to the car in the front:

\[
\frac{dv_n(t)}{dt} = k[V(\Delta x_n(t)) - v_n(t)],
\]

where \( k \) is the sensitivity of a driver and is given by inverse of the delay time.

A calibration of OVM was carried out by Helbing and Tilch to accord to the empirical data and they suggested the form of the optimal velocity function (OVF) as

\[
V(\Delta x_n(t)) = V_1 + V_2 \tanh[C_1(\Delta x_n(t) - L_c) - C_2],
\]

where \( V_1, V_2, C_1, C_2, L_c \) are the fitting parameters.
where \( l_c \) is the length of the cars, which can be taken as 5 m in simulations. The resulting optimal parameter values are \( V_1 = 6.75 \text{ m/s}, V_2 = 7.91 \text{ m/s}, C_1 = 0.13 \text{ m}^{-1}, \) and \( C_2 = 1.57 \). The result of calibration reaches good agreement with the field data, so we still adopt this OVF in this paper.

Although the OVM reveals the density pattern formation of the congested flow of traffic, some endeavors of the model have been done in the case where the optimal velocity function depends only on the headway of each car. One of these improvements is that the next-nearest-neighbor interaction is taken into account.\[7\] Shiro Sawada\[8\] analyzed a generalized optimal velocity model (GOVM). It is assumed that the driver pays attention not only to the headway but also to the headway of the immediately preceding one, thus the formula of GOVM reads

\[
\frac{dv_n(t)}{dt} = k[V(\Delta x_n(t), \Delta x_{n+1}(t)) - v_n(t)], \tag{3}
\]

where

\[
V(\Delta x_n(t), \Delta x_{n+1}(t)) = (1 - p)V(\Delta x_n(t)) + pV(\Delta x_{n+1}(t)), \tag{4}
\]

where \( p \) is a constant ranging from 0 to 1/2, which ensures that the dominant part of the optimal velocity function should be \( \Delta x_n(t) \)-dependent term. In this paper, we adopt the proportional coefficient \( p = 0.2 \).

Comparing GOVM with OVM, we find out that when \( p = 0 \), GOVM has the same form as OVM, and if \( 0 < p < 1/2 \), each driver of a car responds to a stimulus from the preceding car and the next-nearest-neighbor car ahead of him. In general, if the headway \( \Delta x_{i+1}(t) \) of the next car \( i+1 \) ahead is short, the driver of car \( i \) will assume that the forward driver decelerates, thus he will decrease the optimal velocity even though the headway \( \Delta x_i(t) \) is long enough. On the other hand, if the headway \( \Delta x_{i+1}(t) \) of the next car \( i+1 \) ahead is long, the driver of car \( i \) will increase the optimal velocity even though the headway \( \Delta x_i(t) \) is short. Taking the driver’s skill, experience, and psychological effect into account, the GOVM is thus considered to describe traffic flow more realistically.

Applying the linear stability analysis theory, Shiro Sawada investigated the stability of GOVM and obtained that only when the condition

\[
V'(\Delta x_n(t)) < \frac{k}{2}(1 + 2p) \tag{5}
\]

is satisfied, is the traffic stable and robust when small disturbance is added to the uniform traffic flow.

In order to find out the performance of the GOVM, we build a numerical simulation of the motion of cars to check out the model in describing the acceleration and deceleration processes induced by a leading car. First the traffic light is red and all cars are waiting after stop line with headway of 7.4 m, at which the OVF\( (2) \) is zero. Then at time \( t = 0 \), the signal light changes to green and the leading car speeds up to 12 m/s at a rate of 3 m/s\(^2\), which is less than the maximal empirical accelerations of 4 m/s\(^2\).\[6\] We assume that the leading car decreases to the velocity of zero after a period of time of stationary travel. The corresponding speed profile of the leading car is shown in Fig. 1.

![Fig. 1 The velocity profile of the leading car in simulation.](image)

In simulations, we integrate Eq. (3) by the Euler scheme and update the velocity and position of the car

![Fig. 2 Motion of cars 1 ~ 9 according to a pre-specified acceleration and deceleration processes for (a) GOVM and (b) IGOVM. Each curve shows the velocity of each car.](image)
We take the update time interval $\Delta t = 0.1$. The realistic value of delay time in real traffic was investigated by Chandler,[9] who found that it ranges between 1.0 and 2.2. In the simulation we choose a small sensitivity $k = 1$. Since there is only the leading car in front and no next-nearest-neighbor car ahead, the motion of the second car of the platoon obeys the OVM(1). The simulation result is shown in Fig. 2(a). From this figure, we can observe that the GOVM is poor in conducting the simulation process: in acceleration process, the motions of all cars oscillate sharply to the stable state, and unrealistic negative velocities appear in deceleration process.

Why does GOVM not behave well in the aspect? We believe it may be because the model does not consider the velocity difference $\Delta v_n$ on traffic dynamics. We think the $\Delta v_n$ (i.e. the relative velocity $\Delta v = v_{n-1}(t) - v_n(t)$) plays an influential role in traffic dynamics. It is verified by an investigation, performed by a research group of the Bosch GmbH[10], to record follow-the-leader data by means of a floating car. Apart from the vehicle speed and the intervehicle distance, the relative velocity is one of the most significant factors for the description of vehicle dynamics in their experiment. Our observation of real traffic also notes this point.

According to above analysis, we modify the GOVM with consideration of relative velocity. Moreover, random noise is also taken into account in new model, thus the formula of the modified model reads
\[
\frac{d v_n(t)}{dt} = k[V(\Delta x_n(t), \Delta x_{n+1}(t)) - v_n(t)] + \lambda \Delta v_n + \eta \tag{8}
\]
where $\eta$ represents random noise, which is omitted unless otherwise stated in further analytical and numerical exploration of the model. Since the model takes velocity differences into account, we call it an improved GOVM (IGOVM).

Now we apply IGOVM to simulate the car motion above-mentioned. Without loss of generality, here we take $\lambda = 0.5$ and other parameters are the same as before. The results are shown in Fig. 2(b), from which we can clearly observe that all cars obtain stable car-following behaviors and the model fully overcomes the above unrealistic features. From this point of view, IGOVM describes the traffic dynamics more realistically for the case of small delay time, which proves that the improvement in IGOVM is reasonable and correct.

### 3 Existence of Three Traffic Phases and Linear Stability Analysis

Traffic flow is a kind of many-body system of strongly interacting cars. Growing effect has been made in understanding traffic flow dynamics in the last several years. Recent observations[11–15] show that traffic flow demonstrates complex physical phenomena. One of the most important features is the existence of three phases: free flow regime at low densities, coexisting flow regime at intermediate densities, and jammed flow regime at high densities. It has been found out experimentally that the complexity in traffic flow is linked to diverse space-time transitions between the three basically different kinds of traffic.

Next we explore whether the IGOVM can demonstrate the existence of three traffic phases by carrying out a simulation of dynamic evolution of congestion induced by a small perturbation, still taking Eq. (2) for the OVF, $k = 0.5$ and the other parameters being the same as before. Let us take car number $N = 100$, increase circle length $L$ gradually, and set a same initial disturbance as follows:

\[
\begin{align*}
  x_1(0) &= 1 \text{ m}, & x_n(0) &= (n - 1) \frac{L}{N}, & n = 2, \ldots, N, \\
  \dot{x}_n(0) &= V\left(\frac{L}{N}\right) & \text{for} & n = 1, \ldots, N. 
\end{align*}
\]  

![Fig. 3](image3.png) **Fig. 3** Fundamental diagram plot for $\lambda = 0.3$ (dots) and $\lambda = 0.4$ (circles). Dashed line is the homogeneous solution.

![Fig. 4](image4.png) **Fig. 4** Plot of phase for IGOVM for different $\lambda$.

From the simulations we can get the flow-density relation (or the fundamental diagram) of Eq. (8). The simulation result is shown in Fig. 3 as a dotted line, where the
dashed line is the fundamental diagram for the homogeneous flow. It is indicated in Fig. 3 that when the density $k$ is smaller than a critical value $k_1$ or greater than another critical value $k_2$ the flow is the same as the homogeneous solution flow, but when the density falls into the intermediate range ($k_1 < k < k_2$), the measured flow is considerably lower or higher than the homogeneous flow. Therefore, we can define three regimes in traffic flow: free flow ($k < k_1$), coexisting flow regime ($k_1 < k < k_2$), and jammed flow regime ($k > k_2$). In addition, it also should be noted that the coexisting flow range is different with various values of $\lambda$, and $(k_2 - k_1)$ increases with decreasing values of sensitivity, as shown in Fig. 4. The above findings are supported by the analytical results shown below.

In order to check the above analysis, we analyze the linear stability of an $N$-car system on a circular lane of length $L$. It is apparent that the following solution of steady state flow satisfies the dynamical equation (8):

$$x_n^{(0)}(t) = V(b,b)t + nb, \quad b = \frac{L}{N}.$$  \hfill (10)

To see whether the solution (10) is stable or not, we add a small disturbance $y_n(t) = e^{i kn + z t}$,

$$x_n(t) = x_n^{(0)}(t) + y_n(t).$$  \hfill (11)

After neglecting higher terms of $y_n$, the linearized equation is obtained as

$$z^2 + (k - \lambda(e^{ik} - 1))z - kV'(b)((1 - p)(e^{ik} - 1)$$

$$+ p(e^{2ik} - e^{ik})) = 0.$$  \hfill (12)

The solution of Eq. (12) is

$$z = z_1(ik) + z_2(ik)^2 + \cdots$$  \hfill (13)

So the stability condition for IGOVM is given as

$$V'(b) < \frac{k}{2}(1 + 2p) + \lambda.$$  \hfill (14)

Comparing the result with that of GOVM, we can conclude that the improved model is stabilized in the region $(k/2)(1 + 2p) < V'(b) < (k/2)(1 + 2p) + \lambda$ by the effect of introducing the headway of the preceding car.

Qualitative plots of phase for different $\lambda$ are sketched in Fig. 4, from which we have three regimes of stability/instability of the homogeneous flow solution for IGOVM. If $b > b_2$ (free flow) or $b < b_1$ (jam flow) the homogeneous flow solution is stable and if $b_1 < b < b_2$ (coexisting flow) it is unstable, which sufficiently demonstrates why the traffic flow at free and jam flow regimes is the same as the homogeneous solution flow, and the flow at coexisting flow regime is greater or lower than that of the homogeneous flow.

4 Density Waves and Its Stability Analysis

Next we present a more detailed analysis of the dynamical evolution of traffic congestion induced by a small disturbance in density regime of coexisting flow, which is characterized by the presence of humps (dense regions) moving backwards and the evolution of traffic in time and space resembles the propagating density waves when the flow has stabilized. Figure 5(a) presents the snapshots of velocity configuration of all vehicles at $t = 20 000$ s, which is enough simulation time to settle congestion patterns. The conditions of the same simulation as before are: car number $N = 80$, circuit length $L = 2000$ m, sensitivity $k = 0.5$ and the same initial disturbance [Eq. (9)]. We clearly see in the figure that vehicle velocities tend to locate on the two typical values as time develops, which corresponds to the coexisting phase of both jammed and free states as we expect. Furthermore, when the traffic flow becomes stationary after sufficiently large time, the motion of all cars forms a loop as shown in Fig. 5(b).

From the above simulations we also obtain the backward velocity of the congestion and the relation between the density $\rho$ and the flow $Q$ as in Ref. [8]. Considering the bottom end point $(\Delta x_c, v_c)$ and the top end point $(\Delta x_f, v_f)$ in the loop shown in Fig. 5(b), the formula of backward velocity of the congestion reads

$$V_{\text{back}} = \frac{v_f \Delta x_c - v_c \Delta x_f}{\Delta x_f - \Delta x_c}Q - \rho.$$  \hfill (15)
And the relation between the density $\rho$ and the flow $Q$ is given by

$$Q = \frac{v_f - v_c}{\Delta x_f - \Delta x_c} - V_{\text{back}} \rho.$$  \hspace{1cm} (16)

The results are listed in Table 1. From the Table, we learn that $v_c$ is a negative value and $\Delta x_c$ is smaller than the minimum headway 7.4 m when $\lambda = 0.2$ or $\lambda = 0.3$. Bando et al.\[16\] owned it to two possibilities: (i) existence of a new phase and (ii) artificial result for finite-size effects. Moreover, another important feature should be noted that the backward velocity of the congestion increases as $\lambda$ increases.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\Delta x_c$</th>
<th>$v_c$</th>
<th>$\Delta x_f$</th>
<th>$v_f$</th>
<th>$V_{\text{back}}$</th>
<th>$Q(\rho)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>4.386 505</td>
<td>-0.596 036</td>
<td>29.763 116</td>
<td>14.095 269</td>
<td>3.135 51</td>
<td>0.578 93 - 3.135 51 $\rho$</td>
</tr>
<tr>
<td>0.3</td>
<td>6.877 241</td>
<td>-0.109 568</td>
<td>27.288 425</td>
<td>13.620 205</td>
<td>4.735 61</td>
<td>0.672 66 - 4.735 61 $\rho$</td>
</tr>
<tr>
<td>0.4</td>
<td>9.024 810</td>
<td>0.578 107</td>
<td>25.126 747</td>
<td>12.923 223</td>
<td>6.341 08</td>
<td>0.766 69 - 6.341 08 $\rho$</td>
</tr>
</tbody>
</table>

**Table 1** Backward velocity of the traffic congestion and relation between the density $\rho$ and the flow $Q$ for different $\lambda$ in IGOVM.

![Fig. 6](image-url) (a) The plots of velocities for all cars before adding noise; (b) The plots of velocities for all cars after adding noise; (c) Flow converges into the same value before and after adding noise at $t = 10 000$ s.

Our model also demonstrates the phenomenon in empirical investigations that coexisting flow can exist on a highway for a long time, i.e., it can be stable at least with respect to small amplitude noises.\[17\] Simulations are resumed to conduct after the periodic state is stabilized in the above simulation of small disturbance for $\lambda = 0.4$ with an acceleration noise added at $t = 10 000$ s. The results of the simulation are shown in Fig. 6(a) ~ (6c), from which we can observe that the plot of all car velocities before adding the noise is the same as that after adding the noise, and the system is attracted to the same stable limit cycle as before, which adequately illustrates that the stable coexisting flow state of the proposed new model is not sensitive to noise.

5 Sudden Flow Drop and Traffic

Regardless of the cause of congestion, congested traffic usually exhibits two prominent features: (i) an initial front that induces sharp flow change, and (ii) traffic hysteresis, i.e. flow-density plot has loop structure. In 1983, Koshi et al. analyzed traffic data from the Tokyo Expressway and found that flow-density plots resemble “the mirror image of a reversed $\lambda$” (Fig. 7(a)).\[14\] The other important feature, i.e. traffic hysteresis, was provided by
Treiterer and Myers (1974)\textsuperscript{[15]} they found that both flow-density and speed-density plots have loop structures, and Newell (1965) developed a model that contains hysteresis loops (Fig. 7(b)).\textsuperscript{[18]}

By simplifying the analysis a great deal, we conduct a traffic simulation to study these two features of this improved theory on a ring road. It will be helpful to the operation because we do not have to specify boundary conditions. We start with an empty ring road and fill it with car gradually. A car enters or exits from the ring road one by one. An entering car is placed in the middle between two consecutive cars chosen randomly while a leaving car chosen randomly can exit from any location. Our interest is justly in transitions between free and co-existing flow, so we only release 65 vehicles onto the ring road in the aforementioned manner.

Figure 8 shows the simulation results of IGOVM. On the left are the flow-density plots for the entire ring road and on the right are the respective time-velocity plots. It is distinctly noted that the flow-density plot produced shows the desired reversed $\lambda$ image and two major features: (i) flow-drop and (ii) hysteresis loop (arrowed lines). Moreover, another important pattern is verified that the transition from free traffic to coexisting flow occurs at a higher density while the inverse transition occurs at a lower density, which implies that the phase transition from free to coexisting flow is a first-order phase transition.

6 Conclusion

We have proposed the improved car following model of traffic flow for the purpose of fitting the small delay time and extended the GOVM by taking the relative velocity into account. We investigate the effect of the relative velocity on the traffic current and the jamming transition by the use of the numerical and analytical methods with the finding that the new consideration enhances the traffic stability of the traffic flow. The fundamental diagram (the current-density relation) and the phase diagram have been drawn to demonstrate the existence of three phases of traffic flow. Then we apply the improved model to several simulations. The results have demonstrated the potential of the new car-following theory to model complex traffic features of traffic observed experimentally. Furthermore, one point should be noted that the model predicts that the
stable state to relative density in coexisting flow regime is insensitive to noise.

References