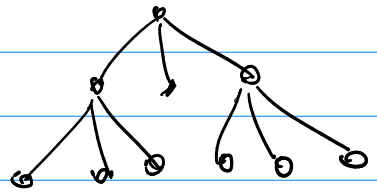
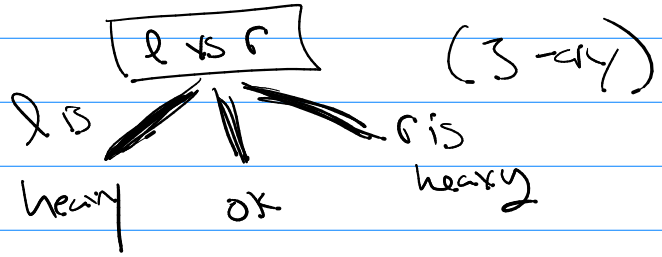
Math 322

Def full 3-ary



Ex 5 coins one may be heavy

① internal vertex / decision

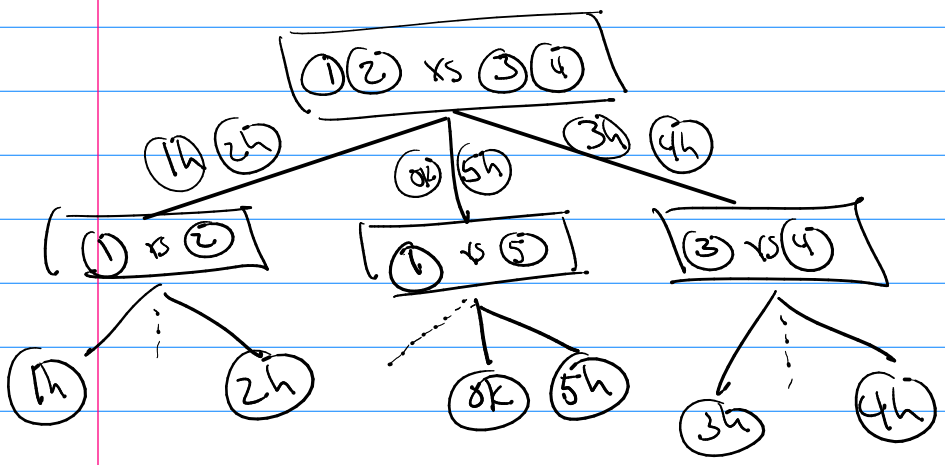


② leaf / outcome

(0h), (1h), (2h), (3h), (4h), or (5h)

③ best case $|\ell| = 6$

$\lceil \log_3 6 \rceil = 2 \leq h$



4 ops

$A \wedge B - A \wedge B - A$

C_4 = # of ways to parenthesize with 4 ops

① $(A) \wedge (B - A \wedge B - A)$ → ways $C_0 \cdot C_3$

C_0 C_3

or

② $(A \wedge B) - (A \wedge B - A)$ ways $C_1 \cdot C_2$

or

③ $(A \wedge B - A) \wedge (B - A)$ ways $C_2 \cdot C_1$

or

④ $(A \wedge B - A \wedge B) - (A)$ ways $C_3 \cdot C_0$

$C_4 = C_0 \cdot C_3 + C_1 \cdot C_2 + C_2 \cdot C_1 + C_3 \cdot C_0$

$C_3 = C_0 \cdot C_2 + C_1 \cdot C_1 + C_2 \cdot C_0$

$C_2 = C_0 \cdot C_1 + C_1 \cdot C_0$

$C_1 = 1$ b/c $(A \wedge B)$

$C_0 = 1$ b/c (A)

Catalan numbers: $C_0 = 1$ $C_1 = 1$ | $C_n = C_0 \cdot C_{n-1} + C_1 \cdot C_{n-2} + \cdots + C_{n-1} \cdot C_0$

$C_2 = 1 \cdot 1 + 1 \cdot 1 = 2$ 1, 1, 2

$C_3 = 1 \cdot 2 + 1 \cdot 1 + 2 \cdot 1 = 5$ 1, 1, 2, 5, 14

→ $C_4 = 1 \cdot 5 + 1 \cdot 2 + 2 \cdot 1 + 5 \cdot 1 = 14$

Language → Grammar

① Natural Language vs Formal Language

② Syntax vs Semantics

Grammar

8.48

sentence → noun phrase, verb phrase

noun phrase → article, adjective, noun

etc.

noun → rabbit

noun → mathematician

the large mathematician hops wildly.

article adj. noun verb adverb

noun phrase verb phrase

sentence

Phrase Structure Grammar

Def ① V is a non-empty set of symbols called a vocabulary or alphabet

② Sentence or word is a finite length string of symbols.