Math 322

Q's? $x + (\bar{y} \cdot (\bar{x} + z))$ only use $+, \bar{}$

$\rightarrow \overline{\square \cdot \triangle} = \bar{\square} + \bar{\triangle}$

$\bar{y} \cdot (\bar{x} + z) = \overline{\bar{y} + (\bar{x} + z)}$

$x + \overline{\bar{y} + (\bar{x} + z)} = x + (y + (\bar{x} + z))$

$x + (\bar{y} \cdot (\bar{x} + z)) = x + \bar{y} \cdot \bar{x} + \bar{y} \cdot z)$

$= (x) + \overline{(y + x)} + \overline{(y + \bar{z})}$

Exam is Friday — 12 probs @ 10 pts

110 pts = 100%

↳ Trees | Boolean Alg. | Q.

11.1 Trees (2 probs)

① $|V| = n$ $\quad |E| = n - 1$

$n = i + l$

Full m-ary $\quad n = mi + 1$

$l \leq m^h \quad \lceil \log_m l \rceil \leq h$

word problem

chain text

m^h

(2) Prove $l \leq m^h$ $h = 1, 2, \ldots$

pf

Basis: (show $l \leq m^h$ for $h = 1$)

$h=1$ tree $\overset{r}{\bullet}$ $c_1 \cdots c_n$ $l = n \leq m$ (n-ary)

$m' = m$

see video for $h=0$

b/c $l = n \leq m = m'$ (true)

Induction: assume $l \leq m^k$ $(h=k)$

show $k+1^{st}$ show $l \leq m^{k+1}$ $(h = k+1)$

→ take up to m-subtrees of height k.

attach to a root. New tree of height $k+1$.

each m subtrees are $l \leq m^k$

total leaves $l \leq \underbrace{m^k + m^k + \cdots + m^k}_{m\text{-subtrees}}$

$l \leq m \cdot m^k = m^{k+1}$ ☐

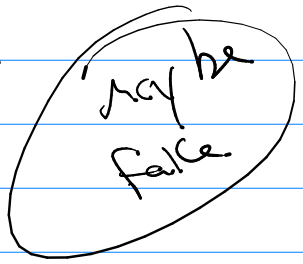
11.2 Apps of Trees (3 probs)

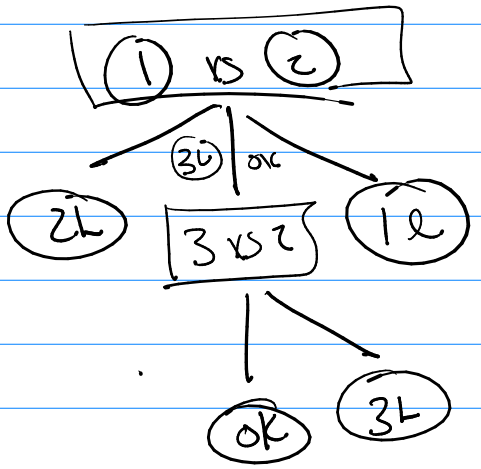
(1) Coin decision tree (ex) 3 coins one is a fake (lighter) find it

(2) Sort

(3) game tree of Nim, tic-tac-toe

3 coins 1 may be light

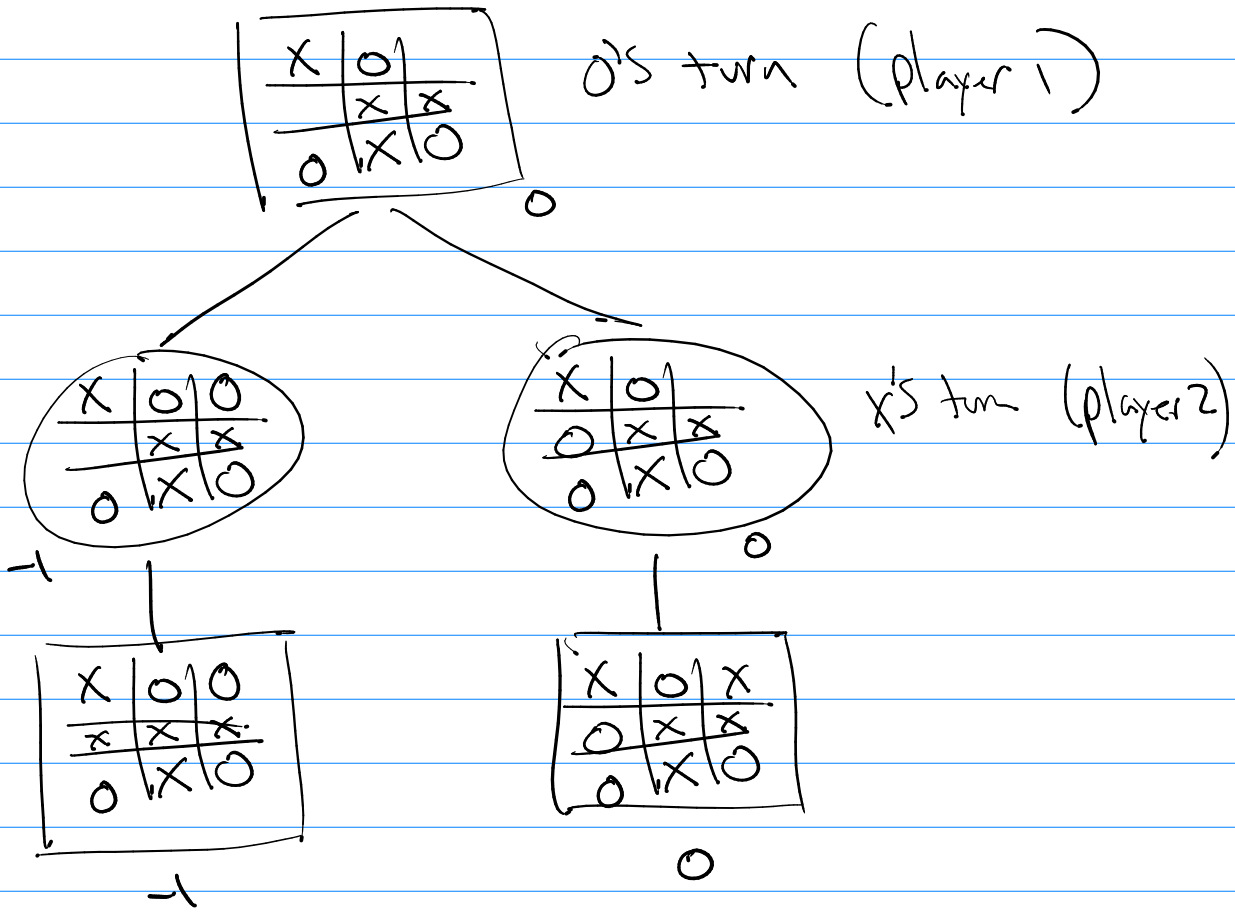




3-ary

$l = 4$

$\lceil \log_3 4 \rceil = 2$



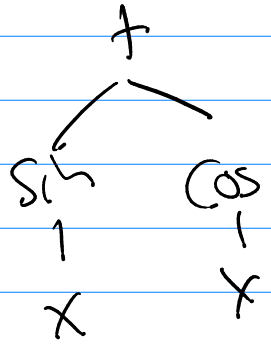
11.3 traversals (3 probs)

① How many ways to parenthesize?

$a+b+c+d$ (Catalan numbers)

② prefix → infix (tree)

postfix



x, sin, +, x, cos in

+, sin, x, cos, x pre

x, sin, x, cos, + post

③ traversals.

12.1 Boolean functions (2 probs)

① Verify laws by bit table

② Use the laws

ex) show $\overline{0} = 1$

$x = x \vee 0$
$= x \vee (x \wedge \overline{x})$
$= (x \vee x) \wedge (x \vee \overline{x})$
$= (x \vee x) \wedge 1$
$= x \vee x$

show $x \vee x = x$

12.2 2 probs

1) sum of products
2) product of sums

Note: be able to do this with or without a table.