Math 322

12.1

p. 815

$\overline{\overline{x}} = x$

$x + x = x$, $x \cdot x = x$

$x + 0 = x$, $x \cdot 1 = x$

$x + 1 = 1$, $x \cdot 0 = 0$

etc

$x + \overline{x} = 1$ unit prop

$x \cdot \overline{x} = 0$ zero prop

} uniq. complements

Verify: ① Table

of $x + \overline{x} = 1$

p q

x	$\overline{x}$	$x + \overline{x}$	1
0	1	1	1
1	0	1	1

Same!

$x + \overline{x} = 1$

② use them

ex college algebra

$x^2 - 2x - 8 = (x-4)(x+2)$

used distrib. law of college Algebra

objects: $\mathbb{R}$

ops: $+, \cdot, -$

ex Boolean Alg.

$$\overline{x \cdot (1 + y)} = \overline{(x \cdot 1) + (x \cdot y)}$$

$$= (\bar{x}) + (x \cdot y) = \bar{x} \cdot (x \cdot y)$$

using these law = "simplification"

$$\overline{x + (y \cdot z)} = \overline{(x+y) \cdot (x+z)}$$

Same as

$$\overline{x \vee (y \wedge z)} = (x \vee y) \wedge (x \vee z)$$

Verify some of the laws given only Identity, Complement, Assoc, Commutative, Distrib.

→ Verify: Idempotent $x \vee x = x$

know

$x \vee 0 = x$, $x \wedge 1 = x$

$x \vee \bar{x} = 1$ $x \wedge \bar{x} = 0$

$(x \vee y) \vee z = x \vee (y \vee z)$, ..

$x \vee y = y \vee x$, ..

$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$, ..

$x = x \wedge 1$

$= x \wedge (x \vee \bar{x})$

$= (x \wedge x) \vee (x \wedge \bar{x})$

$= (x \wedge x) \vee 0$

$= x \wedge x$

$x \wedge x = x$

$$x = x \vee 0 = x \vee (x \wedge \bar{x}) = (x \vee x) \wedge (x \vee \bar{x})$$

$$= (x \vee x) \wedge 1 = x \vee x$$

So $\boxed{x \vee x = x}$

12.2 Representing Boolean Functions

Idea: $F(x_1, x_2, \ldots, x_n)$ = expression of x_i's, 0, 1, and/or $+, \cdot$

Table

x_1	x_2	...	x_n	F
0	0		0	0 or 1
	⋮			⋮
1	1		1	0 or 1

2^n rows → 2^{2^n} total possible tables

So ex

x	y	F
0	0	
0	1	
1	0	
1	1	

$2^4 = 16$ total possible tables.

But $F(x_1, x_2, \ldots, x_n) =$ expression

countably infinite

Given table → find F = expression that makes that table.

ex

x	y	F = expression?
0	0	0
0	1	1
1	0	0
1	1	0

Narrow the type of expressions. (b/c we have an infinite possibility of them)

Terms: ① literal: x or $\overline{x}$

② minterm: product of all the literals

ex $F(x,y,z)$

8 possible minterms xyz, $\overline{x}yz$, $x\overline{y}z$, $\overline{x}\overline{y}z$, $xy\overline{z}$, $\overline{x}y\overline{z}$, $x\overline{y}\overline{z}$, $\overline{x}\overline{y}\overline{z}$

③ Maxterm: sum of all the literal vars.

ex $F(x,y,z)$

8 possible maxterms $x+y+z$, $\overline{x}+y+z$, $x+\overline{y}+z$, $\overline{x}+\overline{y}+z$, $x+y+\overline{z}$, $\overline{x}+y+\overline{z}$, $x+\overline{y}+\overline{z}$, $\overline{x}+\overline{y}+\overline{z}$

Nuki: $a+b+c$

$X+0=X$

$X+1=1$

a	b	c	a+b+c
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	