

Math 322

11.3 Tree Traversals.

(1) Pre-order

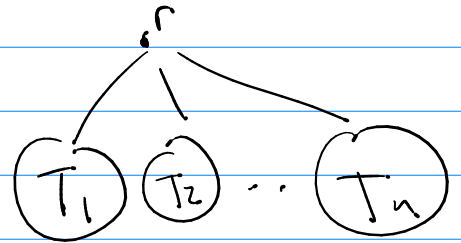
$S_1) r$

$S_2) T_1$

\vdots

$S_{n+1}) T_n$

in pre-order



(2) In-order

$S_1) T_1$

$S_2) r$

$S_3) T_2$

\vdots

$S_{n+1}) T_n$

in In-order

(3) Post-order

$S_1) T_1$

$S_2) T_2$

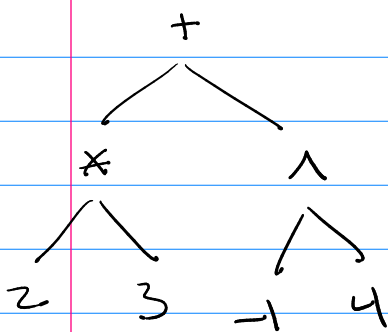
\vdots

$S_n) T_n$

$S_{n+1}) r$

in post-order

Mathematical Operation Tree



pre-order: $+, *, 2, 3, ^, -, 1, 4$

pre-fix notation

In-order: $2, *, 3, +, -, 1, 4, ^$

in-fix notation

post-order: $2, 3, *, -, 4, 1, +, ^$

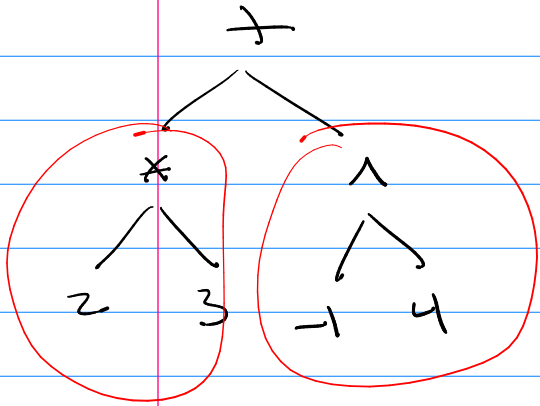
post-fix notation

pre/post fix notation \rightarrow polish / reverse polish notation

Jan Lukiewicz

Properties of pre/in/post-fix notation.

① grouping symbols → grouping subtrees



Infix: $(2, *, 3), +, (-1, ^, 4)$

↑
must have these grouping symbols
= (par is "lost")

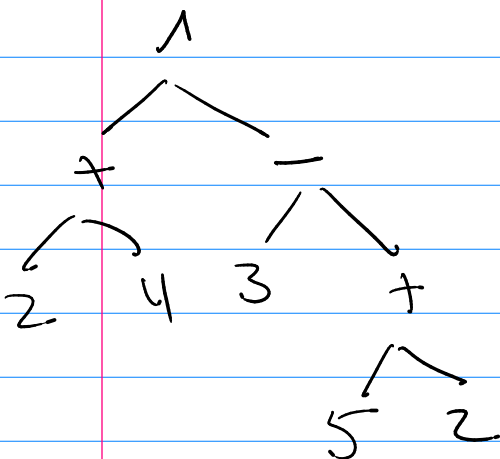
Post-fix $2, 3, *, -1, 4, ^, +$

↓
doesn't need grouping symbols

Pre-fix $+, *, 2, 3, ^, -1, 4$

Notation \Rightarrow tree

$(2 + 4) \wedge (3 - (5 + 2))$



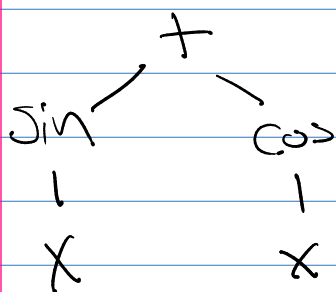
prefix: $^, +, 2, 4, -, 3, +, 5, 2$

postfix: $2, 4, +, 3, 5, 2, +, -, ^$



"typical" Math class notation.

(2x) $\sin(x) + \cos(x)$

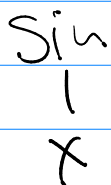


pre-fix: $+ \sin, x, \cos, x$

post-fix: $x, \sin, x, \cos, +$

in-fix: $x, \sin, +, x, \cos$

issue for "typical" notation is unary operators.

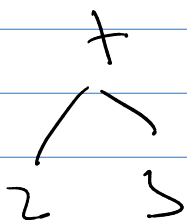


pre-fix: \sin, x ← "typical"

post-fix: x, \sin

in-fix: x, \sin

binary operators



pre-fix: $+ 2, 3$

post-fix: $2, 3, +$

in-fix: $2, +, 3$ ← "typical"

True infix

typical $\sin(x) + \cos(x)$

$x \cdot \sin + x \cdot \cos$

ex) typical $\ln(x) + \sin(e^x)$
true infix $x \cdot \ln + x \cdot \exp \cdot \sin$

Problem with missing grouping symbols for in-fix

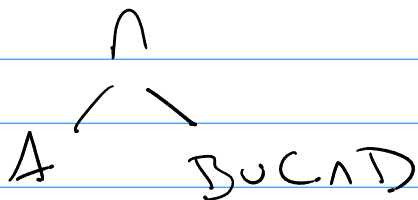
Notation ("lost" the roots)

A N B U C N D

How many ways can we add grouping symbols?

How many ways to tree of this?

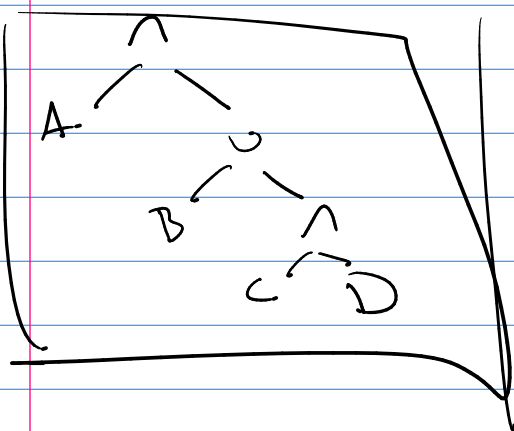
Case 1



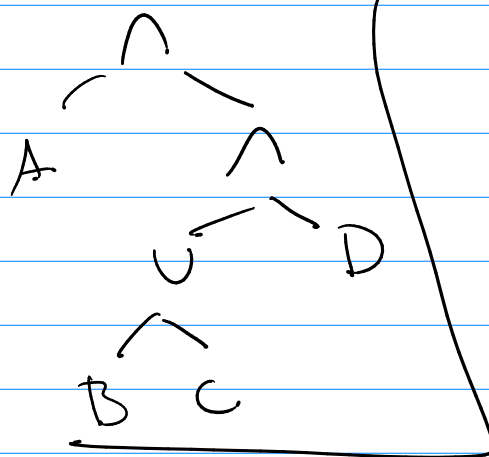
Subcase 1

Subcase 2

#1

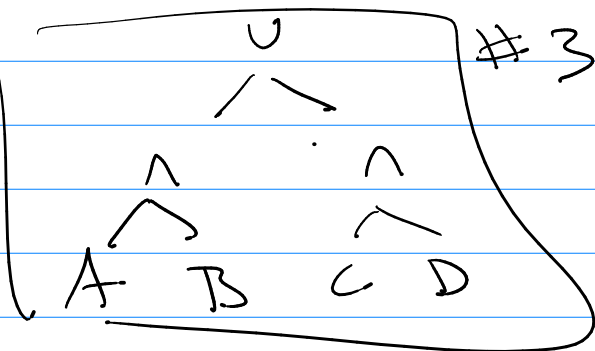


#2

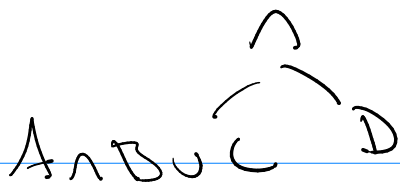


Case 2

#3

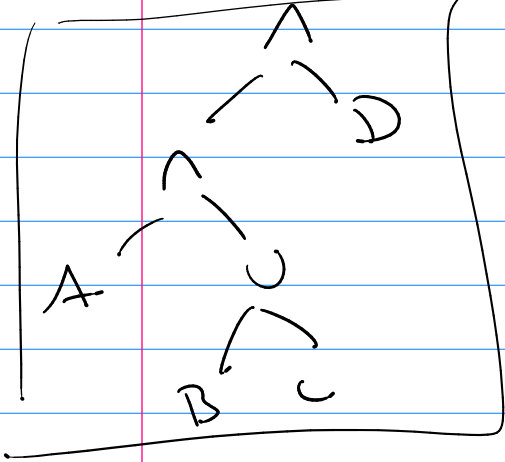


Case 3



Subcase 1

#4



Subcase 2

#5

