Math 322

Trees connected undirected graph with no simple circuits

(Ch 10) **Connected**: simple path between any two vertices

thm tree **iff** unique simple path

pf (tree $\leftrightarrow$ unique simple path)

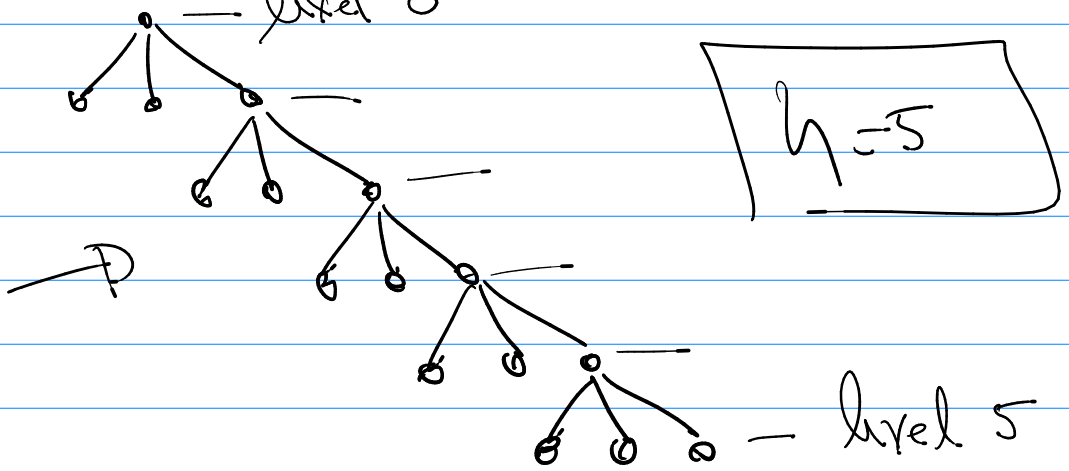
$\equiv$ $\neg$(tree $\leftrightarrow$ $\neg$ unique simple path)

thm $l \leq m^h$

corollary $\lceil \log_m l \rceil \leq h$

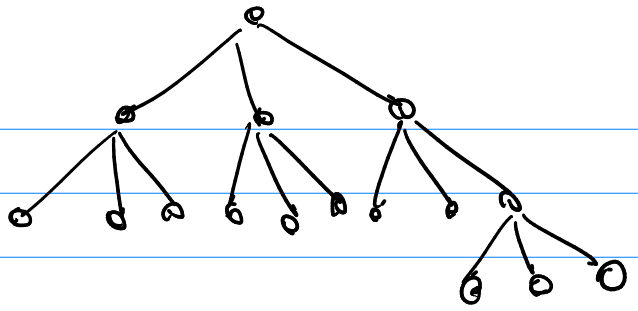
ex Draw a tree with 11 leaves and it is 3-ary

ex



$h = 5$

$\lceil \log_3 11 \rceil \leq 3$



$h = 3$

full and balanced 3-ary tree

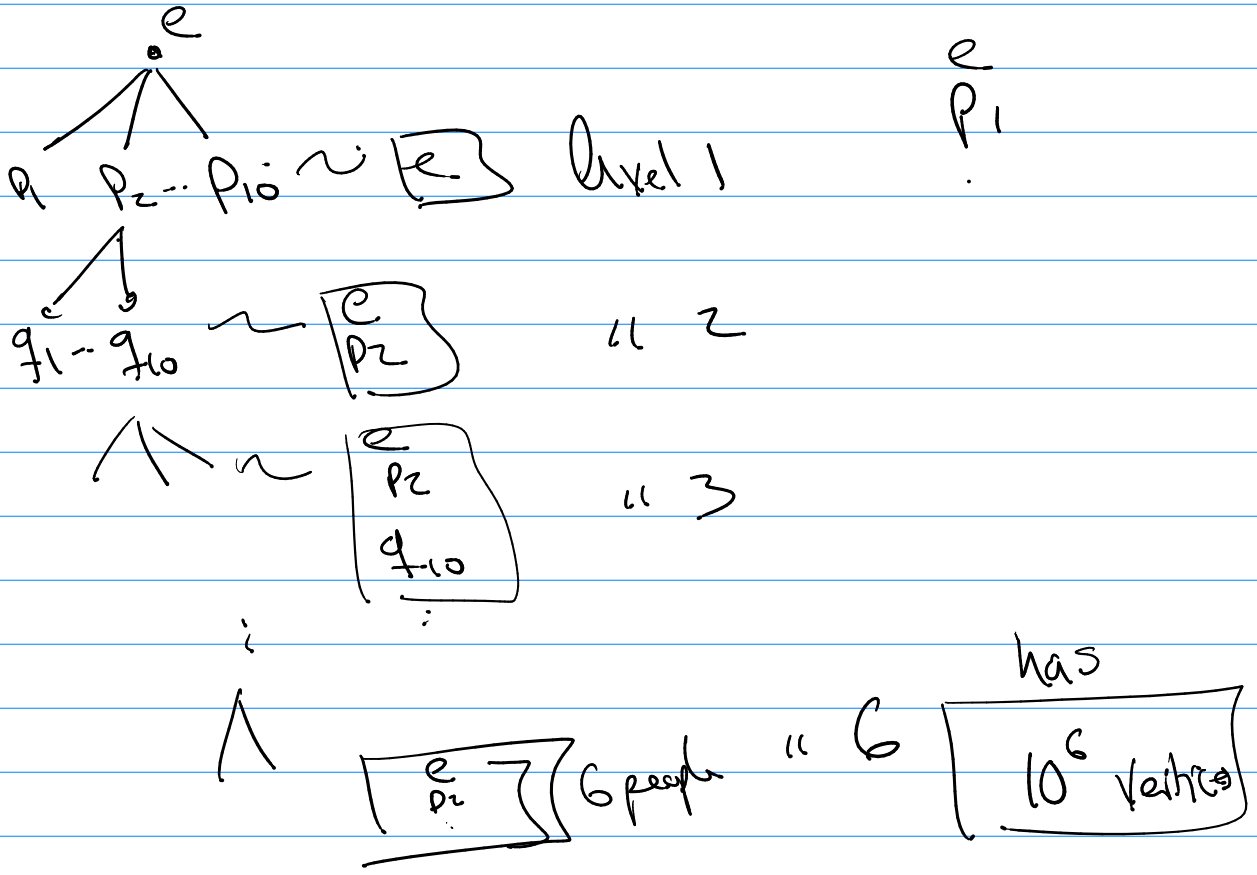
Thm ① total children $m i$ (full n-ary)

② $n = mi + 1$

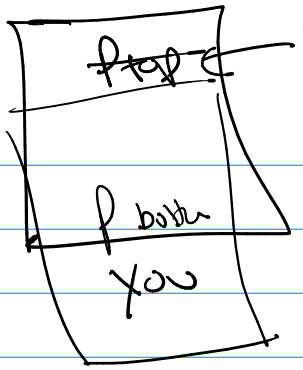
③ $n = l + i$

④ $|V| = n$ then $|E| = n - 1$

Chain letters/text full 10-ary tree



level 7 and on



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assume full-balanced tree...

$c = \$? = \$1{,}000{,}000$ from level 6 to root

10 children & root = \$1,000,000 each from level 7
cost \$10,000,000

100 grandchildren & root = \$1000000 each of level 8
cost \$100,000,000

ex

You get this txt. fwd, fwd, fwd, fwd, fwd - bleh
6 5 4 3 2 1

assume full balanced

level 6 → 10^6 people = 1,000,000

→ skip $l = 1{,}000{,}000$

a) $i = ?_0$ $n = ?_6$

$$\begin{cases} n = i + 1000000 \\ n = 10i + 1 \end{cases}$$

$$0 = 9i - 999999$$

$$i = 111{,}111$$

$$n = 1{,}111{,}111$$

b) $.50 per text → $1 per edge

so $1,111,110 to cell service

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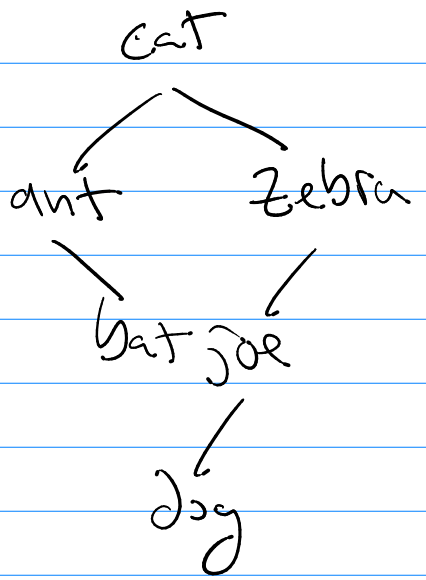
(1) Binary search tree

a) linear chain

words = { ant, bat, cat, dog, joe, zebra }

b) { cat, ant, zebra, joe, dog, bat }



$\lceil \log_2 l \rceil \leq h$

$h = i + 1$

$h = 2i + 1$

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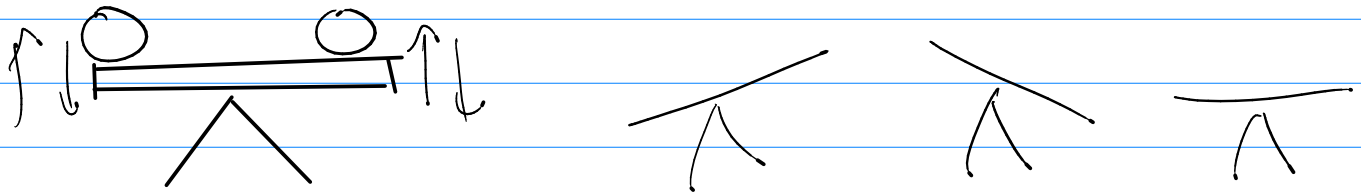
(2) Decision Tree

$i \equiv$ decision to be made

$l \equiv$ outcome of the process

Ex 4 coins and 1 is a fake that is light.

(i) 

(l) (1L) (2L) (3L) (4L) So $l = 4$

Now: b/c $\lceil \log_n l \rceil \leq h$

$\rightarrow \lceil \log_3 4 \rceil = 2 \leq h$