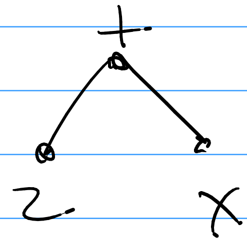
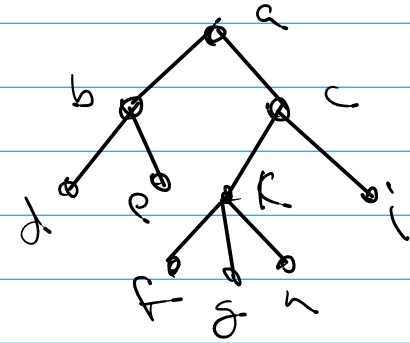
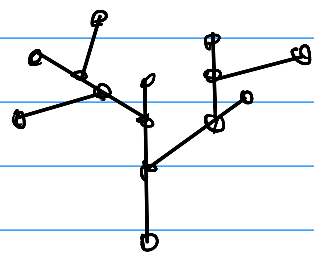
Math 322

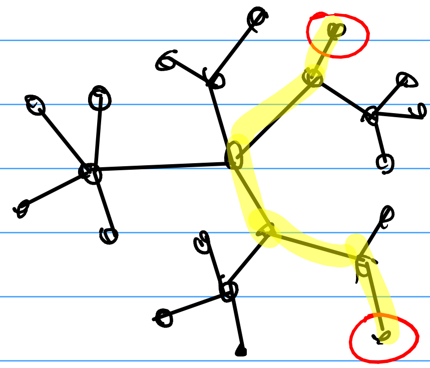
graphs → one 'type'

tree







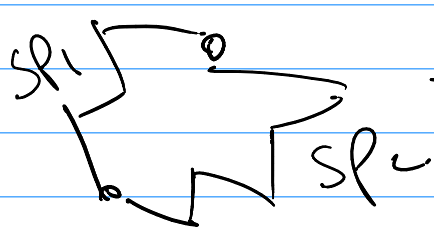


tree ≡ connected undirected graph with no 'null' circuits.

thm G is a tree iff there is a unique simple path between any two vertices.

pf case 1 tree → unique simple path
case 2 unique simple path → tree

case 1 $\neg$ uniq (so at least 2 simple paths) → $\neg$ tree (we have simple circuit)



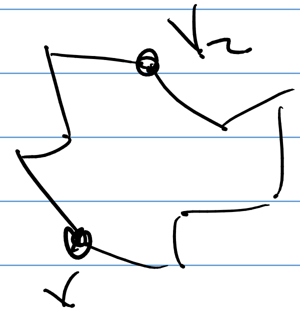
→ we do have a simple circuit.

$\blacksquare$

Case 2 uniq simple path $\rightarrow$ tree

$\equiv$ $\boxed{\neg \text{ tree } \left(\begin{array}{c}\text{have} \\ \text{a simple circuit}\end{array}\right)}$ $\rightarrow \neg$ (uniq.)

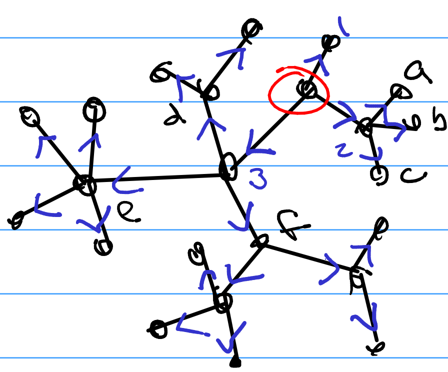
2 or more simple paths

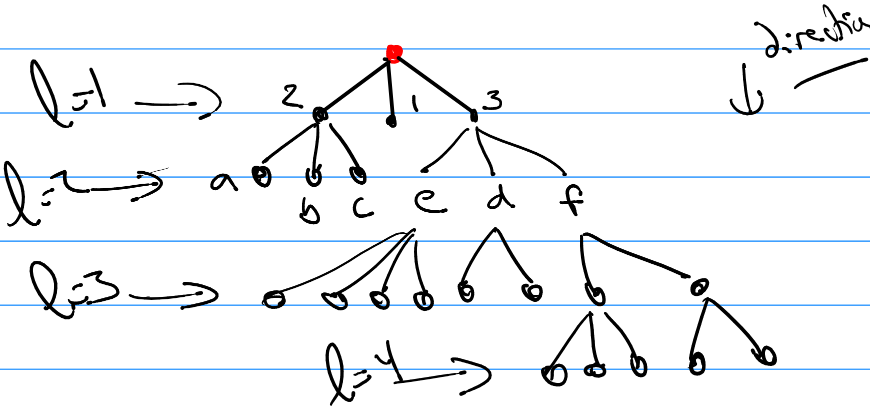


Simple circuit

$\rightarrow$ we have two simple paths to/from v and v_2

Rooted Tree $\equiv$ A tree where we add direction by designating a vertex as the $\boxed{\text{root}}$ and then every edge is directed away from the root.

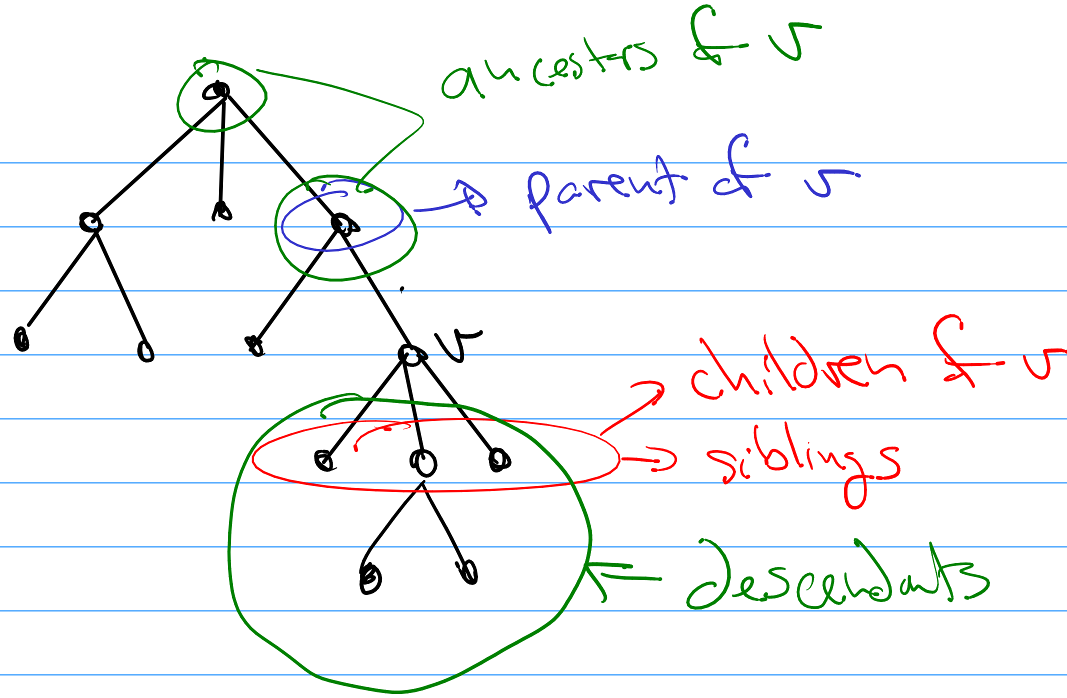


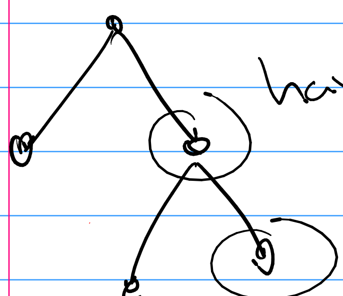


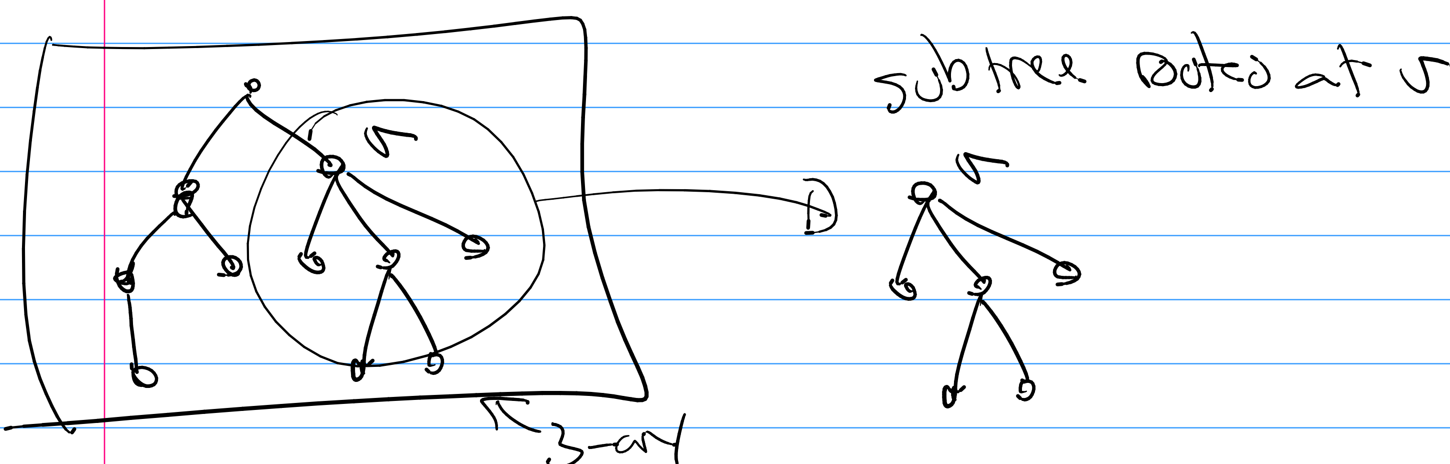
level of a vertex = length of uniq. simple path from root.

height of a rooted tree = max level

terms

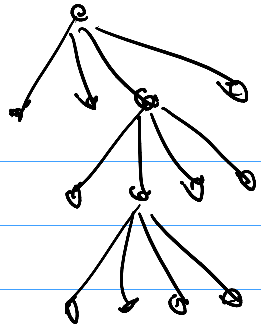






Def:

- ☒ m-ary tree is a tree where every internal vertex has at most m children
- ☒ full m-ary tree is a tree where every internal vertex has exactly m children



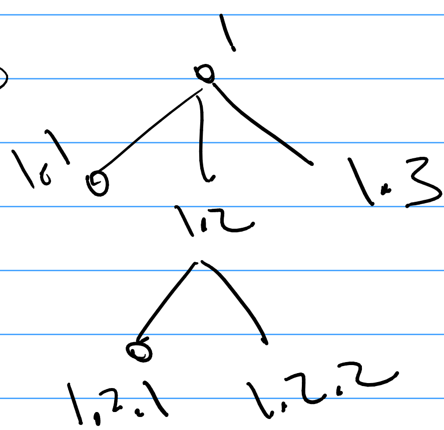
full 4-ary tree

→ **Note:** 2-ary are called **binary**

Def: ordered rooted tree

we apply order to children

ex



Properties:

n is total vertices
l is total leaves
i is total internal vertices

(1) $n = i + l$

(2) Thm full m-ary tree $n = mi + 1$

(3) Thm #4 $\begin{cases} n = i + l \\ n = mi + 1 \end{cases}$

ex full 4 ary tree $\begin{cases} n = i + l \\ n = 4i + 1 \end{cases}$

ex and $i = 10$ $\rightarrow$ $\begin{cases} n = 10 + l \\ n = 4 \cdot 10 + 1 \end{cases}$

$\boxed{n = 41 \quad l = 31}$

Def balanced tree $\equiv$ all leaves are at h (the height of tree) or $h-1$

Th^n $m$ary tree of height h

$\rightarrow \quad l \leq m^h$

Corollary ① $h \geq \lceil \log_m l \rceil$

② full and balanced $h = \lceil \log_m l \rceil$