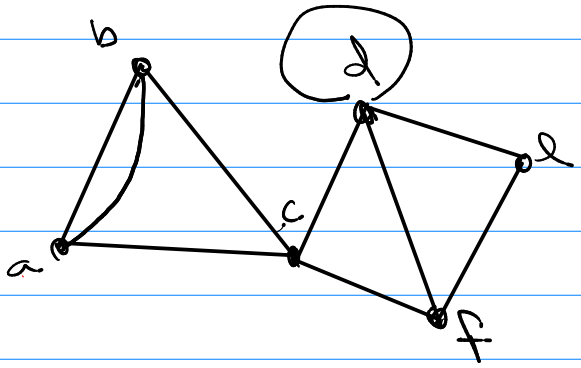
Math 322

10.4 $G = (V, E)$ A is G's adj. matrix

Thm $A^r = [p_{ij}]$

p_{ij} = number of paths of length r from v_i to v_j

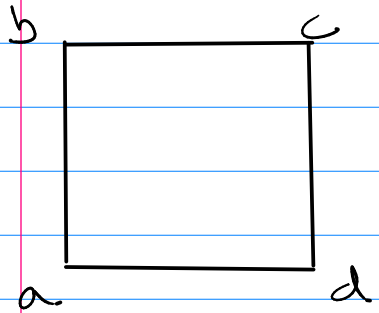
Ex



$$A = \begin{bmatrix} 0 & 2 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & 1 & 2 & 1 & 0 & 1 \\ 1 & 5 & 2 & 1 & 0 & 1 \\ 2 & 2 & 4 & 2 & 2 & 1 \\ 1 & 1 & 2 & 4 & 2 & 3 \\ 0 & 0 & 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 3 & 1 & 3 \end{bmatrix}$$

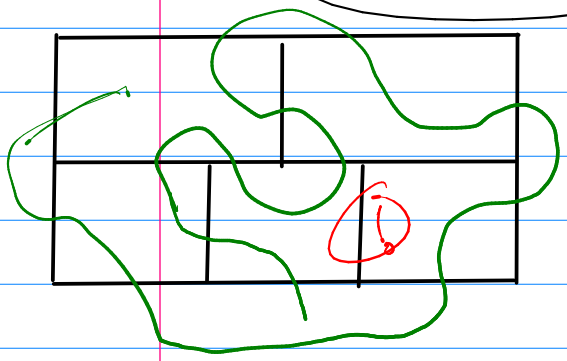
$$A^3 = \begin{bmatrix} 4 & 12 & 8 & 4 & 2 & 3 \\ 12 & 4 & 8 & 4 & 2 & 3 \\ 8 & 8 & 7 & 9 & 3 & 8 \\ 4 & 4 & 9 & 11 & 7 & 8 \\ 2 & 2 & 3 & 7 & 3 & 6 \\ 3 & 3 & 8 & 8 & 6 & 5 \end{bmatrix}$$

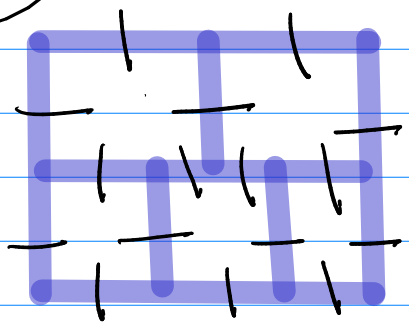


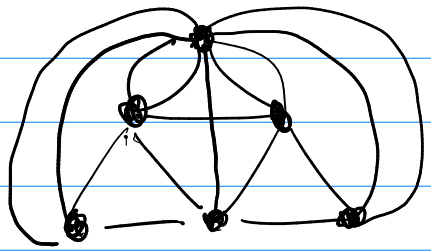
$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

10.5

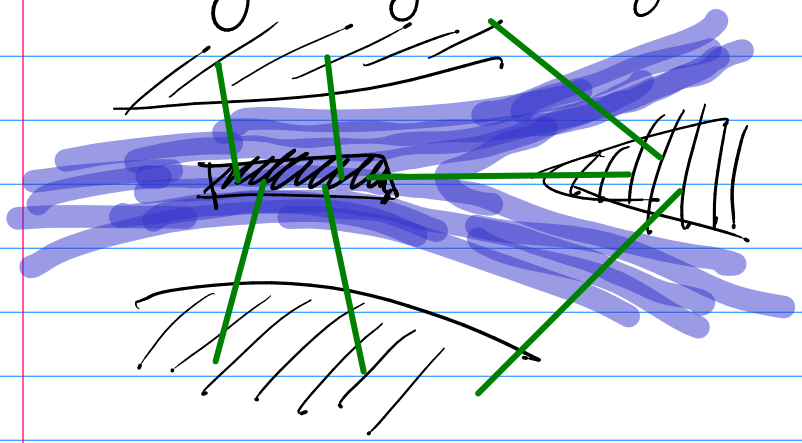
Cut puzzle

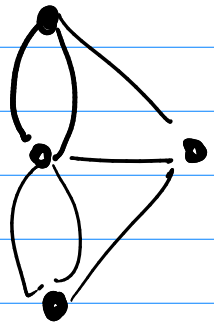






Königsberg Bridge Problem



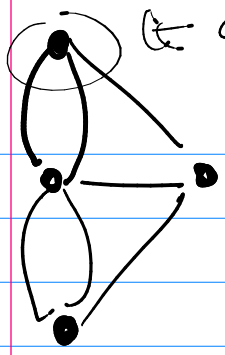


Euler

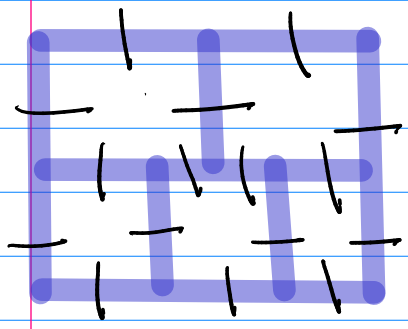
Euler Path, Euler Circuit

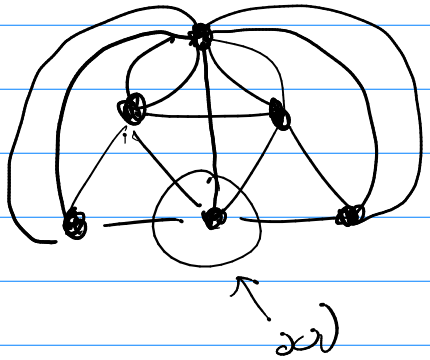
path / circuit that uses every edge exactly once.

Thm Euler circuit iff $\deg(v)$ is even for all $v \in V$.

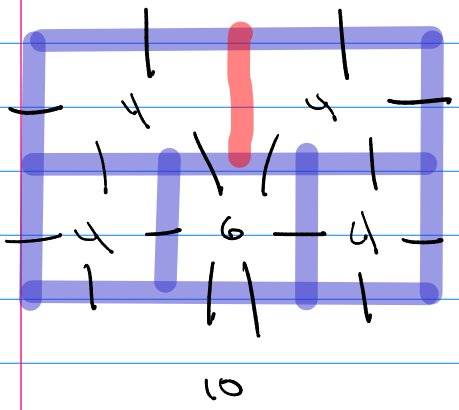


→ no Euler circuit

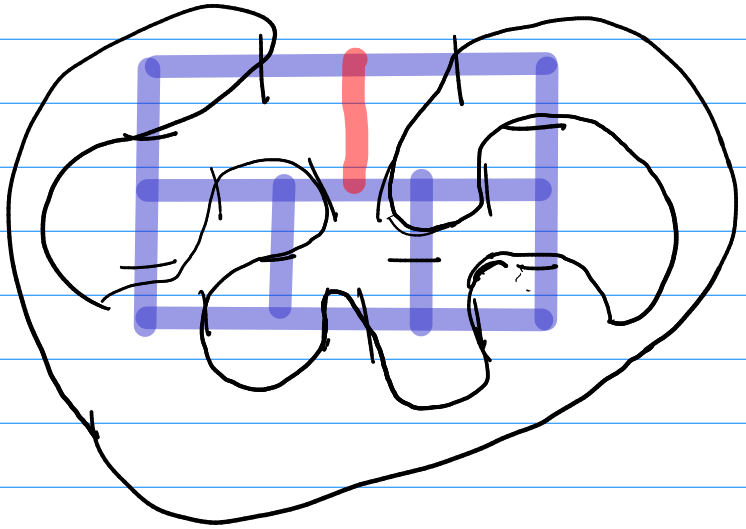


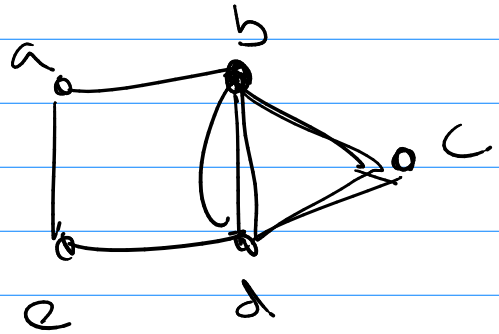


→ no Euler circuit



all even → Euler circuit





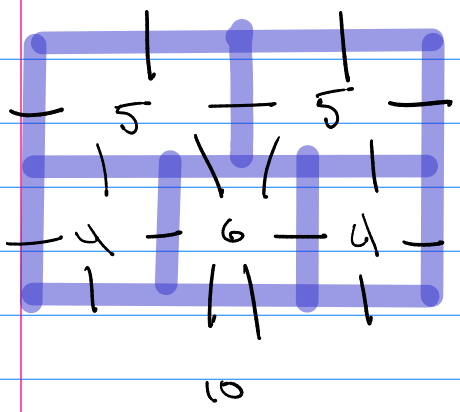
b, c, d, b

b, a, e, d, b, c, d, b

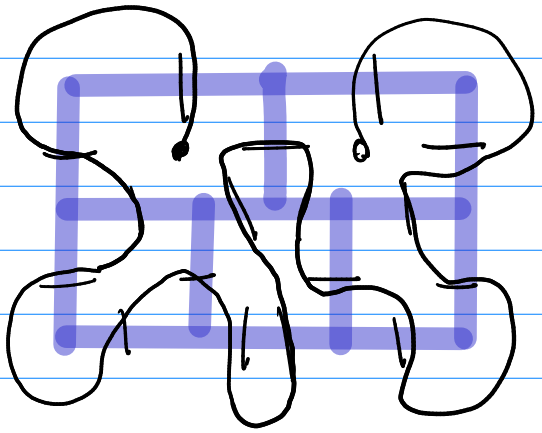
Th 3 Euler path (not circuit)

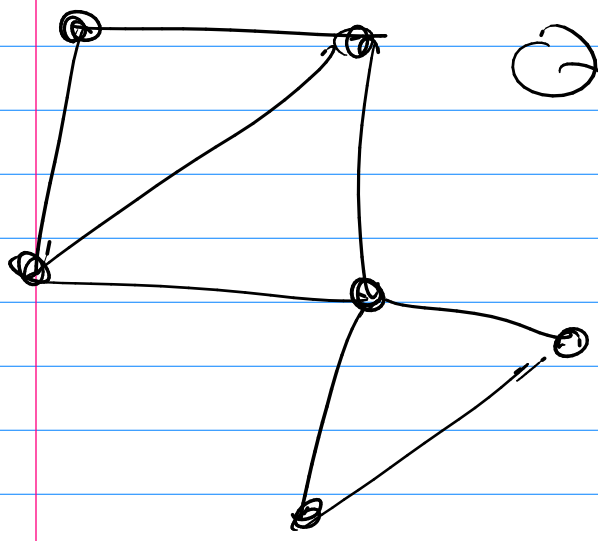
iff deg(v) is odd on exactly two vertices

→ Euler path begins and ends @ the odd vertices.



→ have an euler path





Q: can you use a simple path to visit every vertex exactly once (or if it's a circuit you only visit start = end twice)