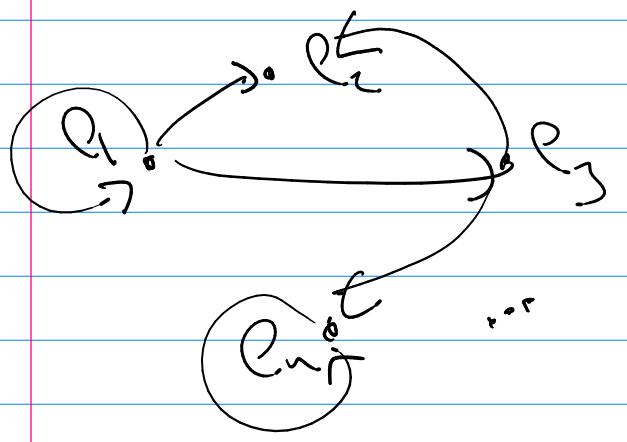
Math 322

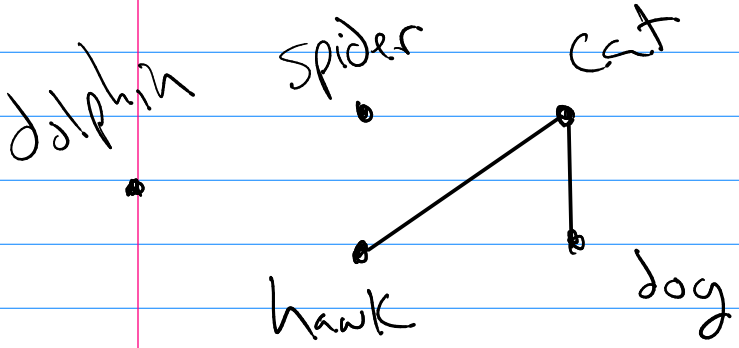
Ch. 10 Graphs

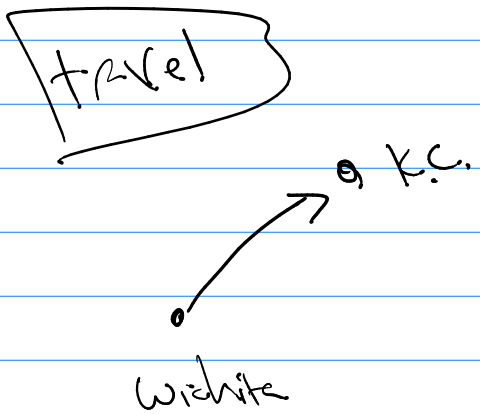
$R = \{(a,b) \mid a,b \text{ satisfy a propositional function}\}$ on set $A = \{e_1, e_2, \ldots, e_n\}$

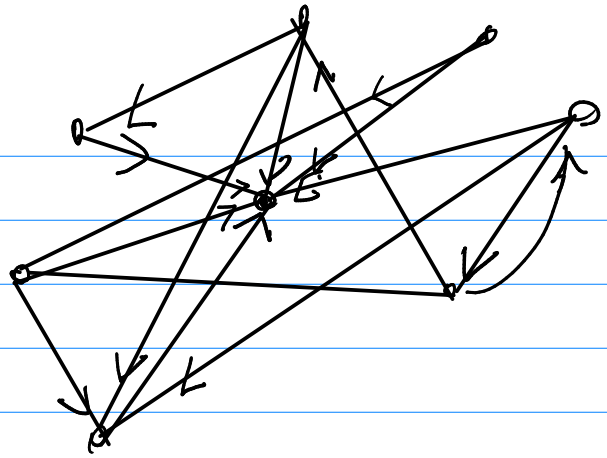


digraph

Model biological competition ← same food source





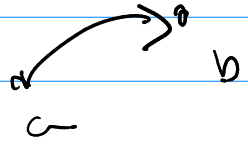


Graphs $G = (V, E)$

V = non-empty set of vertices
E = set of edges

Directed Graph (E)

$e \in E$ is an ordered pair
$e = (a, b)$

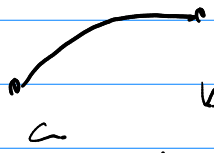


a is adjacent to b
b is adjacent from a

a is the initial vertex of e
b is the terminal vertex of e
b is the end vertex of e

Undirected Graph (E)

$e \in E$ is an unordered pair
$e = \{a, b\}$



a, b are adjacent
a, b are neighbors
e connects a, b

$N(v) = \{a \mid a, v \text{ are neighbors}\}$
neighborhood of v.

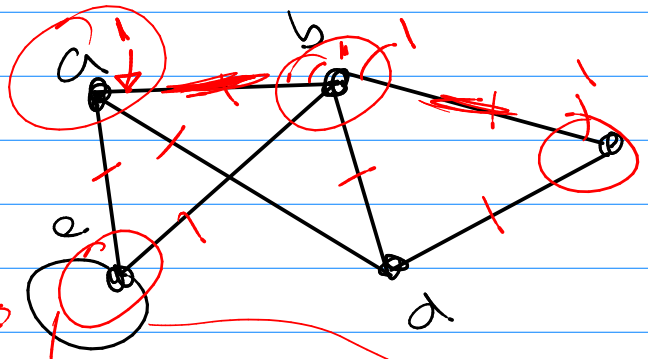
$$N(A) = \bigcup_{x \in A} N(v)$$

Count connections

Degree

Undirected: $\deg(v)$ = # of edges with v as a vertex, except loop count for 2.

ex



$\{a,e\}$
$\{e,e\}$
$\{e,b\}$

$|E| = 8$

$\deg(a) = 3$
$\deg(b) = 4$
$\deg(c) = 2$
$\deg(d) = 3$
$\deg(e) = 4$

$2 \cdot |E| = 16$

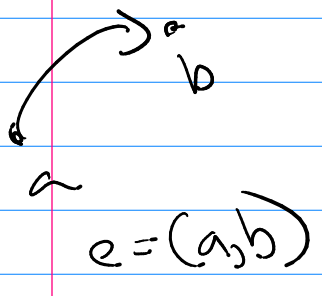
Thm $\sum_{v \in V} \deg(v) = 2|E|$

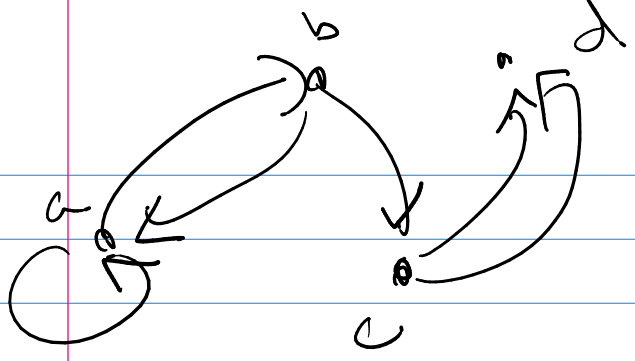
Thm even number of vertices with odd degree.

Directed

out degree: $\deg^+(v)$ = # of edges with v as initial vertex

in degree: $\deg^-(v)$ = # of edges with v as term. vertex





$|E| = 6$

$\deg^+(a) = 2$ $\quad \deg^-(a) = 2$

$\deg^+(b) = 2$ $\quad \deg^-(b) = 1$

$\deg^+(c) = 2$ $\quad \deg^-(c) = 1$

$\deg^+(d) = 0$ $\quad \deg^-(d) = 2$

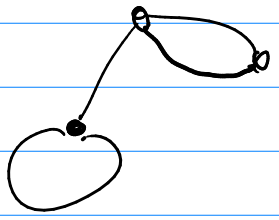
6 $\qquad$ 6

Thm $\sum_{v \in V} \deg^+(v) = \sum_{v \in V} \deg^-(v) = |E|$

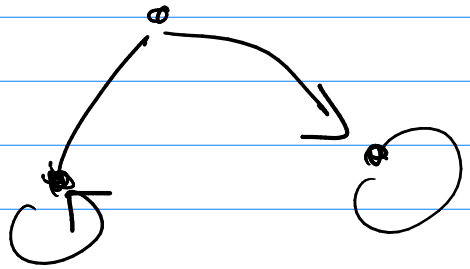
Types of Graphs

Undirected	Directed
(1) Simple Graph → no loops → no mult. edges	(1) Simple directed graph → no loops → no mult. edges
(2) Multi Graph → no loops → allows mult. edges	(2) directed multigraph → allows loops → allows multi edges

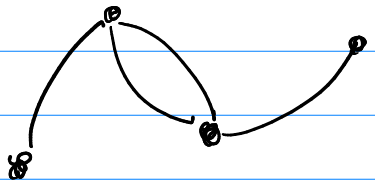
(3) Pseudo graph
→ allow loops
→ allow multi edges



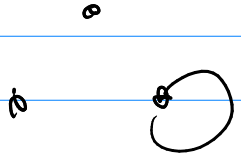
Mixed Graph $E = \{$ undirected and directed pairs $\}$



Ex

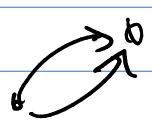


Multigraph



pseudo graph

simple directed graph



directed multigraph