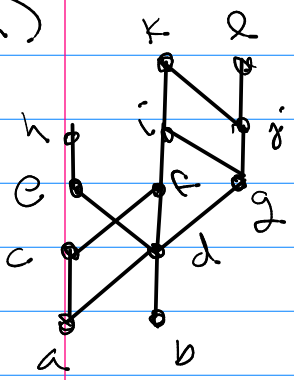
Math 322

Extremal Properties

$(S, \preceq)$



Minimal / Maximal (not nec. to be uniq)

① e is a minimal if
$\equiv$
$\neg \exists b \, (b \prec e)$
$\equiv \forall b \, (b \not\prec e)$

minimals $= \{a, b\}$

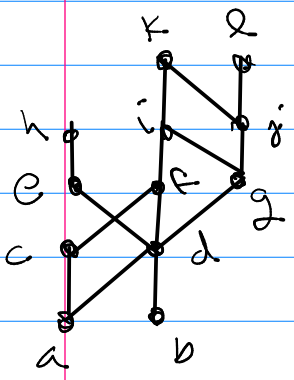
② e is a maximal if
$\equiv$
$\neg \exists b \, (e \prec b)$
$\equiv \forall b \, (e \not\prec b)$

maximals $= \{h, k, l\}$

Uniq. extremals (if they exist)

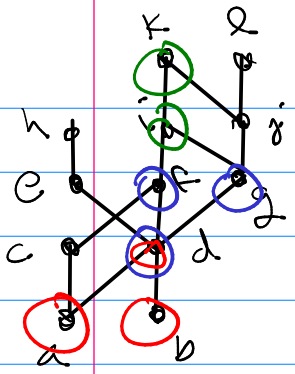
① e is the greatest if $\forall b \, (b \preceq e)$

② e is the least if $\forall b \, (e \preceq b)$



greatest $= \{\ \}$

least $= \{\ \}$



given a subset, A, of $(S, \leq)$

(1) u is a upper bound of A if

$\forall a \in A \ (a \leq u)$

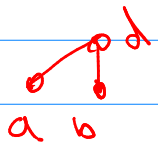
(2) l is a lower bound of A if

$\forall a \in A \ (l \leq a)$

ex) $A = \{d, f, g\}$

upper bounds $= \{i, k\}$

lower bounds $= \{a, b, d\}$



(3) least of the upper bounds

least upper $= \{i\}$

(4) greatest of the lower bounds

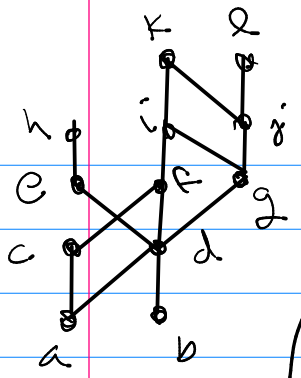
greatest $= \{d\}$

Topological Sort

$(S, \leq)$ is a poset (partial) ← some elements do not compare

→ add edges to $(S, \leq)$ so the new relation is a chain.

and if $a \leq b$ in $(S, \leq)$ they are still $a \leq b$ in new.



topological Sort:

take a minimal from the Hasse diagram and remove it and its edges from the diagram

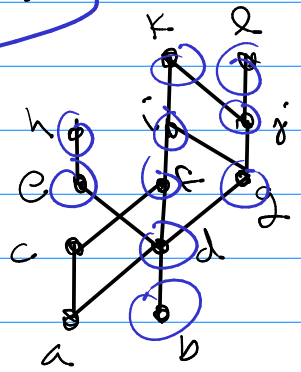
loop

ex1 $a < c < b < d < e < h < g < j < l < f < i < K$

ex2 $b < a < d < g < j < l < c < f < i < K < e < h$

(take the right minimal)

ex3 (left) $a < c < b < d < e < h < f < g < i < j < K < l$



Exam 1 — 13 probs @ 10 pts

→ 120 pts = 100%

Ch 9

9.1 Relations and Properties (3 probs)

9.2 n-ary relations (Databases) (1 prob)

9.3 Representing Relations (2 probs)

9.4 Closures (3 probs)

9.5 Equiv. Relations (2 probs)

9.6 Partial orderings (2 probs)

9.1 Know: all 6 properties (logical form)

ex: trans: $\forall a \forall b \forall c (aRb \wedge bRc \rightarrow aRc)$

$\equiv$ "

Know: eqs

Know: th^ #1 p. 580 - 581

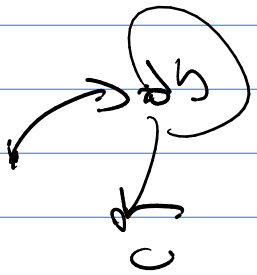
trans iff $\forall n R^n \subseteq R$

9.2 Database

Know: Keys, join

9.3 Representation

Know: M_R = [zero-one matrix] properties

Know: digraph 

Know: ops as matrices

9.4 Closures

Know: all 3 closures.

Note: on transitive closure know

- $M_{R^*} = M_R \vee M_R^{[2]} \vee \cdots \vee M_R^{[n]}$
- Warshall's

9.5 Equiv.

Know - properties?

equiv. classes?

9.6 Posets

Know - properties?

Hasse Diagram

Extremals

Topo Sort.