Math 322

Q's $p \quad q \quad p \oplus q$ $\qquad S \circ R : A \to C$

$R : A \to B$

$S : B \to C$

Thm R is transitive iff $\forall n \; R^n \subseteq R$, $n = 1, 2, 3, \ldots$

pf fact $\square \leftrightarrow \triangle$

tech #1 $\square \equiv$ step 1 $\equiv$ step 2 $\equiv \ldots \equiv \triangle$

tech #2 case 1 $\square \to \triangle$

case 2 $\triangle \to \square$

Facts: R is trans : $\forall a \forall b \forall c \, (aRb \wedge bRc \to aRc)$

$R^n \subseteq R \to R^n = R^{n-1} \circ R$

$aR^n c \quad \exists b \; aRb \wedge b R^{n-1} c$

$R^n \subseteq R \equiv aR^n c \to aRc$

$(a,c) \in R^n \to (a,c) \in R$

Pf case 1 $\forall n \; R^n \subseteq R \to R$ is transitive

case 2 R is transitive $\to \forall n \; R^n \subseteq R$

Proof of case 1 "$\forall n \; R^n \subseteq R \to R$ is transitive"

Sketch 2 assume $R^n \subseteq R$ $\to$ show $\to$ R is transitive

$aR^n c \to aRc$ $\qquad$ $aRb \wedge bRc \to aRc$

$\star$

$a R^n c \to a R c$

$\boxed{(a R b \wedge b R^{n-1} c) \to a R c}$

same for $n=2$ it is true for all n

(end of scratch paper)

assume $R^n \subseteq R \equiv [a R^n c \to a R c]$

let $n = 2$ (univ. inst)

$\boxed{\to} [a R^2 c \to a R c]$

$\equiv [a R b \wedge b R c \to a R c] \equiv R$ is trans.

$\boxed{\text{prove case 2}}$ "R is trans $\to \forall n \; R^n \subseteq R$, $n = 1, 2, \ldots$" ⊠

assume R is trans. now $\boxed{\text{show } \forall n \; R^n \subseteq R, \; n = 1, 2, \ldots}$

$(a R b \wedge b R c \to a R c)$

or $\boxed{\text{induction}}$

$\boxed{\text{Induction}}$ Basis Step: show $R^1 \subseteq R$ knowing R is trans.

$R \subseteq R \equiv T$ ⊠

Inductive show [k^{th} step $\to$ $k+1^{st}$ step

$(R^k \subseteq R \to R^{k+1} \subseteq R)$

assume the inductive hyp.

$(\boxed{\square R^k \Delta} \to \boxed{\square R \Delta})$ $(\square R^{k+1} \Delta \to \square R \Delta)$

$\boxed{a R^{k+1} c} \equiv a (R^k \circ R) c \equiv [a R b \wedge b R^k c]$

$\xrightarrow{\text{by IH}} [a R b \wedge b R c] \xrightarrow{\text{b/c R is trans}} \boxed{a R c}$ ⊠

$\boxed{9.2}$ n-ary relations : Application = Relational Database

R is an n-ary relation (subset of $A_1 \times A_2 \times \dots A_n$)

Def: n : degree of R

A_i : domains

Relational Database

Table	:	n-ary relation
Database	:	set of n-ary relations
field	:	A_i domain
record	:	n-tuple $(a_1, a_2, .., a_n)$

→ if a single domain can be used to uniq. find a record → that is a $\boxed{\text{primary key}}$

→ if you need $A_{i_1} \times A_{i_2} \times .. \times A_{i_m}$ fields to uniq. find a record → $\boxed{\text{composite key}}$

→ Tables change in time.

at any moment in time ; table is an extension of a relation

the parts of R that are unchanging : intension of a relation

Ops

(1) take some records of a table $\rightarrow$ new table (Selection)

S_c maps R to n-tuples from R that match C.

S_c
$\uparrow$
condition of selection

(2) take some fields of all records (projection)

$P_{i_1, i_2, \dots, i_m}$ takes m-tuples using fields $i_1, i_2, \dots, i_m$

(3) table 1 + table 2 $\rightarrow$ New table (Join)

R deg m $\quad$ S deg n

$J_p(R,S)$ is $m+n-p$ tuples

from S

$$(a_1, a_2, \dots, a_{m-p}, c_1, c_2, \dots, c_p, b_1, b_2, \dots, b_{n-p})$$

from R

p = # of similar fields
