# Math 344

~~Q's~~ ?

**Final Exam:**

→ Combine Exams 1–3 (33)

$\boxed{\text{1 hr 50 mins}}$

→ $\boxed{\text{6 probs / exam}}$

**Note:** Calc 1/2 Integration skills

→ power, trig, sum/diff, substitution, $e^x$, $\frac{1}{x}$
parts, $\int ?\, dx = \operatorname{atan}(x) + C$

Diff. Skills

→ power, trig, chain, prod/div, sum/diff,
$e^x$, $\ln(x)$

$[\operatorname{atan}(x)]' = ?$

tangent

Exam 1

? 0

1) a) Find the point of the curve $\boldsymbol{r}(t) = \langle t^3+1, t+1, \sin(t)\rangle$, $t \in [0, \pi/2]$ where the tangent line is parallel to the y-axis.

(Find $\vec{r}\,' = 0$ $\qquad \vec{r} = \langle P, Q, R\rangle$

$\vec{r}\,' = \langle P', Q', R'\rangle$

b) Evaluate the integral.

$$\int \langle 2t^2+t, t\sin(t^2), \ln(t)\rangle dt$$

$\int \vec{r}\,dt = \langle \int P\,dt, \int Q\,dt, \text{etc}\rangle$

2) Find the length of the curve $\boldsymbol{r}(t) = \left\langle 1, \frac{1}{2}t^2, \frac{1}{3}t^3\right\rangle$, $t \in [0, 1]$.

? 0

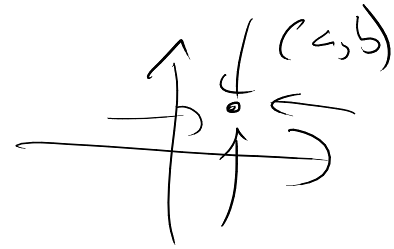
3) Find the velocity and position vectors given the acceleration function $\boldsymbol{a}(t) = \langle 2, 6t, 12t^2\rangle$ with initial values of $\boldsymbol{v}(0) = \langle 1, 0, 0\rangle$ and $\boldsymbol{r}(0) = \langle 0, 1, -1\rangle$.

$\vec{r} \underset{\text{Int}}{\overset{\text{deriv}}{\rightleftarrows}} \vec{v} \underset{\text{Int}}{\overset{\text{deriv}}{\rightleftarrows}} \vec{a}$

? 0

4) Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{2y^4\cos^2 x}{x^4+y^4}$$




5) a) Find the three first partial derivatives of the function $f(x, y, z) = x\sin(y^2+z) + y$. Do not simplify your answers.

$f_x$ $\quad$ 2nd partials $\quad f_{xx} \quad f_{yy} \quad f_{zz}$

$f_y \qquad\qquad\qquad f_{xy} \quad f_{yx} \quad f_{zx}$

$f_z \qquad\qquad\qquad f_{xz} \quad f_{yz} \quad f_{zy}$

b) Find the partial derivative $H_{xxy}$ of the function $H(x, y, z) = yx^2 + y^2x^3 + x\sin(z)$

6) The radius and height of a closed cylindrical are measured as 2cm and 10cm, respectively, with an error in measurement of at most 0.1cm in each. Use differentials to estimate the maximum error in the calculated volume where $V = \pi r^2 h$.

? 0

7) If $w(x, y, z) = xyz$, $x(s, t) = s + \cos(t)$, $y(s, t) = s + \sin(t)$, and $z(s, t) = st$, find $w_s$ and $w_t$. Do not simplify your answers.

? 0

8) Find the directional derivative of $f(x, y) = e^x \sin(y)$ at $(0, \pi/2)$ in the direction of $\langle -6, 8\rangle$.

9) Find all the critical points of $f(x,y) = xe^{-2x^2-2y^2}$ and classify the relative extrema.

10) A rectangular box without a lid is to be made from $12\text{cm}^3$ of cardboard. Setup the system of equations that would find the maximum volume of such a box using the method of Lagrange multipliers. ~~(Do not solve the system of equations)~~

11) The pressure $P$ (in kilopascals), volume $V$ (in liters), and temperature $T$ (in kelvins) of a mole of an ideal gas are related by the equation $PV = 8.31T$. The pressure of 1 mole of an ideal gas is increasing at a rate of 0.05 kPa/s and the temperature is increasing at a rate of 0.15 K/s. Find the rate of change of the volume when the pressure is 20 kPa and the temperature is 320 K.

Same as 8 (Note: a problem of the class of 8, 9 will be on exam)

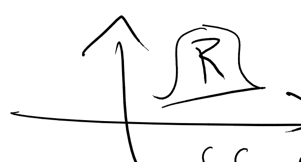
EXAM 2

1) Calculate the double integral

$$\iint_R \frac{x}{1+xy}\, dA$$

over the region $R: 0 \le x \le 1, 0 \le y \le 1$. Hint: the inner integral should be with respect to $y$.

2) For the double integral




$$\iint_R xy\, dA$$

$\iint_R f\, dA$

over the region $R$ enclosed by $y = x^2$ and $y = 4$ setup ~~(DO NOT EVALUATE)~~ the iterated integral ...

a) in order of $dx \cdot dy$

b) in order of $dy \cdot dx$

3) Evaluate the integral by changing to polar coordinates

$dA = r\,dr\,d\theta$

$$\iint_R \sin(x^2+y^2)\, dA$$

where region $R$ is in the first quadrant between the circles centered at the origin of radii 1 and 3.

Center of mass?

?
0 4) Find the mass of a rectangular lamina with vertices (0,0), (2,0), (0,3) and (2,3) if the density function is $1+x+y$.

5) Find the moment about the x-axis ($M_x$) for the rectangular lamina with vertices (0,0), (2,0), (0,3) and (2,3) if the density function is $1+x+y$.

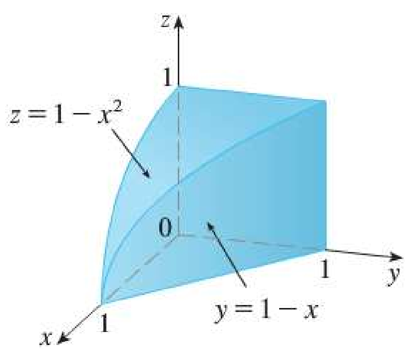
6) Evalute the triple integral

$$\iiint_E xy + z^2 \, dV$$

$dV = dx\,dy\,dz$

where $E: 0 \le x \le 1, 0 \le y \le 2, 0 \le z \le 3$.

7) Setup the triple integral for a function $f(x,y,z)$ with respect to the given region

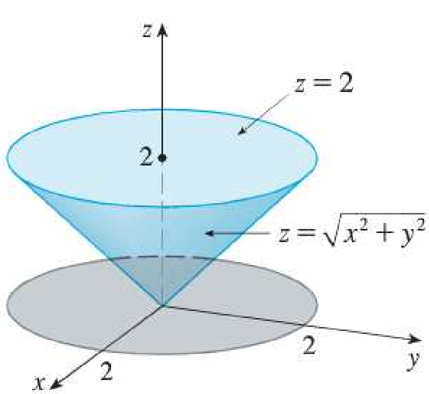



in the order of $dz \cdot dy \cdot dx$

?
0 8) Evaluate the integral of $(x^2+y^2)$ within the given 3D region using cylindrical coordinates.

or

Spherical




$dV = r\,dr\,d\theta\,dz$

$dV = \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$

9) Let $E$ be the portion of a solid sphere centered at the origin of radius 2 above the plane $z = 1$ whose density at any point is proportional to its distance from the origin. Find the center of mass of $E$.

?

10) Verify the double integration formula for polar coordinates by using the change of variables in a double integral formula (basically use the Jacobian).

11) The boundary of a lamina consists of the semicircles $y = \sqrt{1-x^2}$ and $y = \sqrt{4-x^2}$ together with the x-axis that join them. Find the center of mass if the density at any point is inversely proportional to its distance from the origin.

Same class as #4