Math 344

Wed → Take Home Exam Ch 13

11 probs → 110 pts

→ 100 pts = 100%

Q 13.7

$$\iint_{S_{xyz}} f(x,y,z)\, dS = \iint_{D_{uv}} f(\boxed{\vec{r}(u,v)})\, |\vec{r}_u \times \vec{r}_v|\, dA$$

parameters for S_{xyz}

S: is an explicit function $z = g(x,y)$

parametric form
$x = x$
$y = y$
$z = g(x,y)$

Ch 13 (11 probs)

13.1 $\vec{F}(x,y,z)$ Vector fields

(0 probs)

13.2 (2 probs) Line Integrals

① $\int_C f\,ds$ $\qquad \int_C f\,dx$ $\qquad \int_C f\,dy$

C — $\boxed{\text{parametric form}}$ $\vec{r}(t)$ $\quad a \le t \le b$

$ds = |\vec{r}\,'(t)|\,dt$ $\qquad dx = x'\,dt$

$dy = y'\,dt$

$\Rightarrow \int_a^b f(\vec{r})\,\boxed{|\vec{r}\,'|\,dt}$

ex $\int_C f\,ds$

① give C (parametric?)

② give f

→ Setup (5pts) $\qquad ds$

$$\int_a^b \boxed{\underbrace{f(\vec{r}(t))}_{\text{f is now in t's}}\ \overbrace{|\vec{r}\,'|}\,}dt$$

② line integral on $\vec{F}$

$$\int_C (\vec{F} \cdot \vec{T})\,ds = \int_a^b \boxed{\left(\vec{F}(\vec{r}) \cdot \vec{r}\,'\right)}dt$$

$\vec{T}$ ← unit tangent along C

13.3 Fund. Thm for line Integrals

(1 prob)

$$\int_C \underbrace{(\nabla f)}_{\vec{F}} \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

ex) $\vec{F} = \langle P, Q \rangle$

if $P_y = Q_x \rightarrow \vec{F} = \nabla f$

$\rightarrow \begin{cases} P = f_x \\ Q = f_y \end{cases}$ Solve for f

So) $\vec{F} = \langle xy^2, x^2y \rangle$

a) <u>Conserv?</u>

$\left.\begin{array}{l} \frac{\partial}{\partial y}(xy^2) = 2xy \\ \frac{\partial}{\partial x}(x^2y) = 2xy \end{array}\right\}$ <u>yes</u>

$\begin{cases} \boxed{f_x = xy^2} \rightarrow f = \int xy^2\,dx \\ \boxed{f_y = x^2y} \end{cases}$

$f = \frac{1}{2}x^2y^2 + C(y)$

not x's

$f = \frac{1}{2}x^2y^2 + C(y)$

$f_y = x^2y + C'(y)$

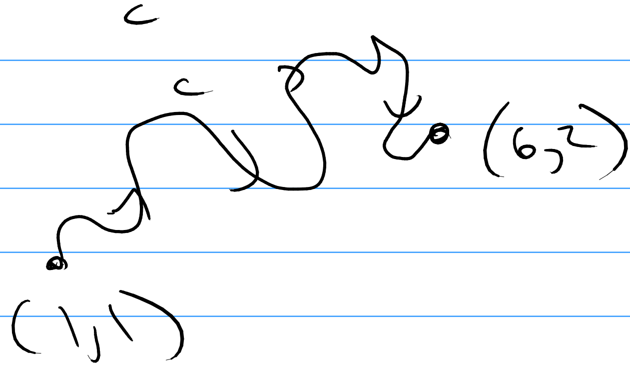
Must be equal

so $c'(y) = 0$

so $c(y) = C$

$$\boxed{f = \frac{1}{2}x^2y^2 + C}$$

$$\Rightarrow \int_C \langle x^2y, xy^2\rangle \cdot d\vec{r} = f(@\,b) - f(@\,a)$$



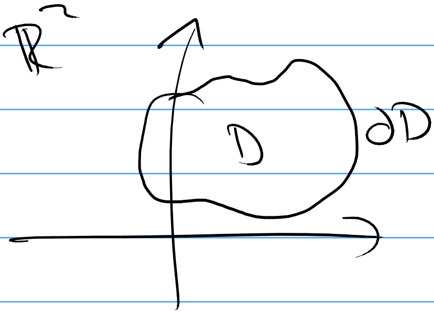
$$= \frac{1}{2}(6)^2(2)^2 - \frac{1}{2}(1)^2(1)^2$$

$$= \frac{1}{2}(4\cdot 36 - 1)$$

$$= \boxed{\frac{143}{2}}$$

13.4 Green's 1 prob

$\mathbb{R}^2$



$$\int_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA$$

ch 12

13.5 Curl / Div (1 prob)

1 Do them.

$\text{curl}(\vec{F}) = \nabla \times \vec{F}$

$\text{div}(\vec{F}) = \nabla \cdot \vec{F}$

13.6 Parametric Surf (2 probs)

for S ... (given parametric form
or finding parametric form)

1 A(S)

2 Tangent Plane to S

13.7 Surface Integrals (2 probs)

1 $\iint_S \underbrace{f}_{\text{scalar}} \, dS$

2 $\iint_S \vec{F} \cdot d\vec{S} = \iint_S \underbrace{(\vec{F} \cdot \vec{n})}_{\text{scalar}} \, dS$

normal to $\vec{S}$

13.8 Stokes's (1 problem)

① $$\underbrace{\int_C \vec{F} \cdot d\vec{r}}_{\text{find?}} = \iint_S \text{curl}(\vec{F}) \cdot d\vec{S}$$

13.9 Div Th^n (1 problem)

① $$\iint_S \vec{F} \cdot d\vec{S} = \underbrace{\iiint_E \text{div}(\vec{F})\, dV}_{\text{ch 12}}$$