Math 344

13.5 $\vec{F} = \langle P, Q, R \rangle$

Operations:

① curl($\vec{F}$) $\qquad$ curl : $\vec{F}_1 \to \vec{F}_2$

Ⓐ $\text{curl}(\vec{F}) = \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle$

$$\nabla(f) = \left\langle \frac{\partial}{\partial x} f, \frac{\partial}{\partial y} f, \frac{\partial}{\partial z} f \right\rangle$$

$$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

Ⓑ $\text{curl}(\vec{F}) = \nabla \times \vec{F}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

ex $\vec{F} = \langle \overset{P}{yzx}, \overset{Q}{z\sin(x)}, \overset{R}{xz} \rangle$

$\text{curl}(\vec{F}) = \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle$

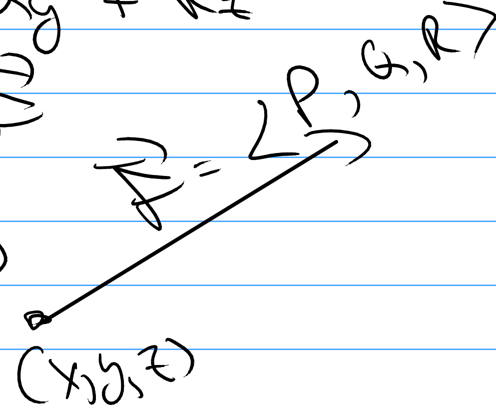
$= \langle 0 - \sin x, yx - z, z\cos(x) - zx \rangle$

② **Divergence:** $\operatorname{div} : \vec{F} \to \mathbb{R}$

$$\operatorname{div}(\vec{F}) = \boxed{P_x} + Q_y + R_z$$

$$\operatorname{div}(\vec{F}) = \nabla \cdot \vec{F}$$

$$\operatorname{div}(\vec{F}) = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle P, Q, R \rangle$$



ex) $\vec{F} = \langle xyz, \sin(y^2)z, x\tan(z) \rangle$

$$\operatorname{div}(\vec{F}) = P_x + Q_y + R_z$$

$$= yz + 2yz\cos(y^2) + x\sec^2(z)$$

Properties

$$\operatorname{curl}(\vec{F}) = \nabla \times \vec{F}$$
$$\operatorname{div}(\vec{F}) = \nabla \cdot \vec{F}$$

① if $f(x,y,z)$ has cont 2^{nd} order partials

$$f_{xx}, f_{xy}, f_{xz}$$
$$f_{yx}, f_{yy}, f_{yz}$$
$$f_{zx}, f_{zy}, f_{zz}$$

$$\to \operatorname{curl}(\underbrace{\nabla f}_{\vec{F}}) = \boxed{\nabla \times (\nabla f)} = \vec{0}$$

So when we used Fund. th$^{\underline{m}}$ of line Integrals...

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

$\boxed{\text{but}}$ we know $\vec{F}$ is conserv. vector field

$\exists f$ such that $\vec{F} = \nabla f$

potential functn

$\boxed{3D}$ $\boxed{\text{Check}}$ $\text{curl}(\vec{F}) = \vec{0}$

$\Rightarrow \vec{F}$ is a conserv. vector field.

(You can apply fund. th$^{\underline{m}}$ after finding f)

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(2) $\boxed{\text{Th}^m}$ $\vec{F} = \langle P, Q, R \rangle$ (cont 2$^{\underline{nd}}$ order partials)

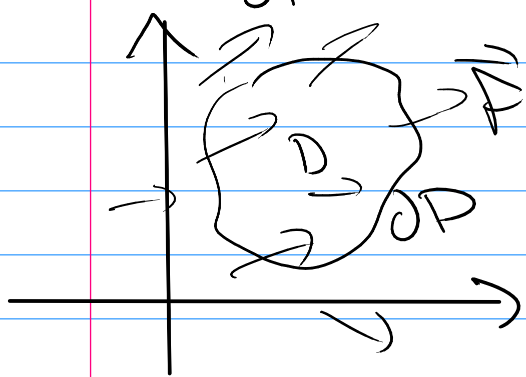
$$\Rightarrow \text{div}(\text{curl}(\vec{F})) = 0$$

Application: $\boxed{\text{do}}$ $\text{div}(\vec{H}) = 0$ ← fine

so $\vec{H} = \text{curl}(\vec{F})$ ← must equal; ← some vector field

These will also give us two new notations for Green's thm:

$$\int_{\partial D} \vec{F}\cdot d\vec{r} = \int_{\partial D} P\,dx + Q\,dy = \iint_{D} \underline{\underline{(Q_x - P_y)}}\,dA$$



$\boxed{2D} \xrightarrow{\text{make}} (3D)$

$\vec{F} = \langle P, Q, 0 \rangle$

$$\operatorname{curl}(\vec{F}) = \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & 0 \end{vmatrix}$$

$$= \langle 0, 0, \boxed{Q_x - P_y} \rangle$$

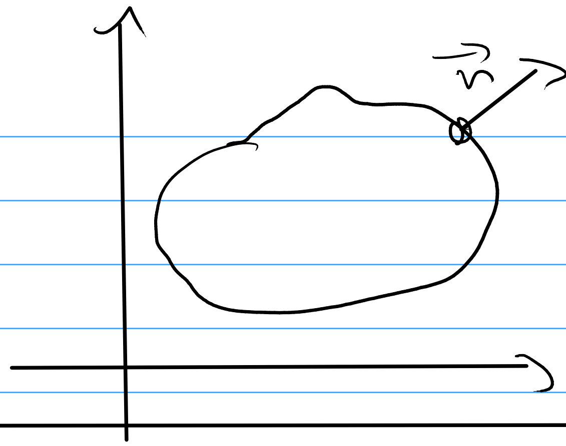
$$(\nabla \times \vec{F})\cdot \vec{k} = (Q_x - P_y)\,\vec{k} \cdot \vec{k}$$

Compare: $\displaystyle \iint_{D} (Q_x - P_y)\,dA$

$$\boxed{\int_{\partial D} \vec{F}\cdot d\vec{r} = \iint_{D} \left(\operatorname{curl}(\vec{F})\cdot \vec{k}\right) dA}$$

Vector Notation using curl for Green's Thm

Similarly



$$\vec{n} = \frac{1}{|\vec{r}'|}\langle y', -x' \rangle$$

$$\boxed{\int_{\partial D} \vec{F} \cdot \vec{n}\, ds = \iint_D \operatorname{div}(\vec{F})\, dA}$$

Vector Notation using div

of Green's Th^m^