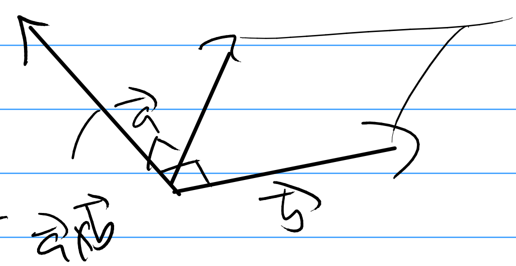
Math 344

Q's of Exam 2

Note: These problems (some & them) are on the final!

Do Not just put exam away!

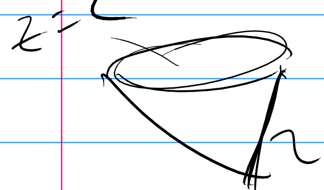
Tackle: idea



$$|\vec{a}\times\vec{b}|$$

$$\iiint_E (x^2+y^2)\,dV \qquad \text{Cylindrical } (r,\theta,z)$$

$$\iiint r^2 \, r\,dr\,d\theta\,dz$$



$z=\sqrt{x^2+y^2}$

$z=r$

$$\int_0^{2\pi}\left(\int_0^2\left(\int_r^2 r^3\,dz\right)dr\right)d\theta$$

= Finish

ch 13 Vector Calculus

3D $\vec{F} = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$

$\vec{F} : \mathbb{R}^3 \to \langle \;,\;,\; \rangle$ in 3D

line integral

$$\int_C f \, ds$$ f scalar

$\vec{F}$ vf

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{F} \cdot \vec{T} \, ds = \int_a^b \left(\vec{F}(\vec{r}) \cdot \vec{r}\,' \right) dt$$

Fixed on 11/12

$$= \int_C P\,dx + \int_C Q\,dy + \int_C R\,dz$$

$$= \int_C P\,dx + Q\,dy + R\,dz$$

$$= \int_a^b P(\vec{r}(t))\,x'\,dt + \int_a^b Q(\vec{r}(t))\,y'\,dt + \int_a^b R(\vec{r}(t))\,z'\,dt$$

(given $\vec{F}$)

Note: if f a scalar function exists such that $\nabla f = \vec{F}$

Call $\vec{F}$ conservative vector field

f is the potential function of $\vec{F}$

(Calc 1) **Note:**

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

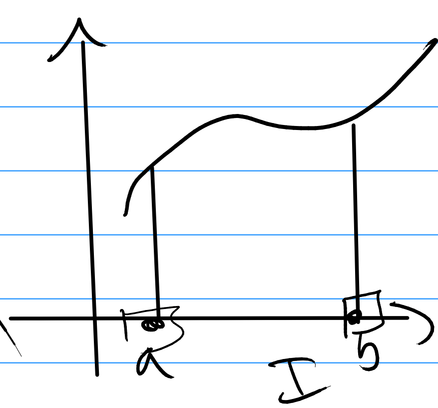
and $\frac{d}{dx}\left[F(x) + C\right] = f(x)$

Diff Notation

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Fund thm of Calculus

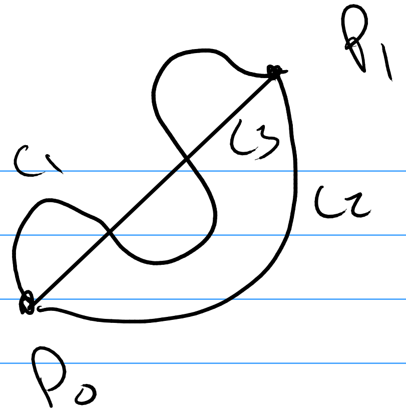
$f(x) = F'(x)$



$$\int_I F'(x)\,dx = F(b) - F(a)$$

endpts of I

(Q) Is there something like this for $\vec{F}$?



Idea: $\displaystyle\int_{C_1 \text{ or } C_2 \text{ or } C_3} \vec{F} \cdot d\vec{r} = ?$

doesn't depend on C_1, C_2, rather P_1, P_2

---

th^m given $\vec{F}$ and there is a f such that $\nabla f = \vec{F}$ ($\vec{F}$ is a cons. vector field w/ f potential func)

$$\to \boxed{\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))}$$

can only apply this on conservative vector fields

Q #1 is $\vec{F}$ a conservative vector field?

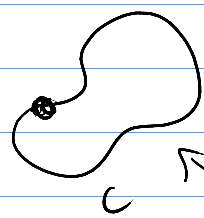
know: $\displaystyle\int_C \vec{F} \cdot d\vec{r} = \left(f(\vec{r}(b)) - f(\vec{r}(a))\right)$

path doesn't matter

only depends on $f(\vec{r}(b))$ $f(\vec{r}(a))$ (end pts)

Thm $\int_C \vec{F}\cdot d\vec{r}$ is ind. of path in D

iff $\int_C \vec{F}\cdot d\vec{r} = 0$ for every C that is closed in D.



closed

Thm $\vec{F}$ is cont. in D (open and connected)

if $\int_C \vec{F}\cdot d\vec{r}$ is ind. of path

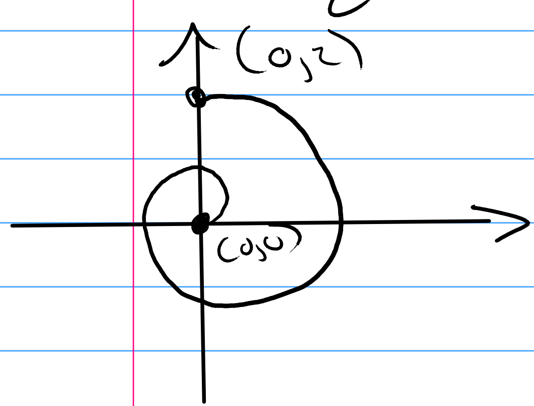
$\Rightarrow$ $\vec{F}$ is a cons. vector field.

Thm **2D** $\vec{F} = \langle P, Q \rangle$

if $P_y = Q_x \rightarrow \vec{F}$ is cons. vector field.

---

ex $\int_C \vec{F}\cdot d\vec{r}$ $\quad \vec{F} = \langle y + \cos x, x \rangle$



before: $C: \langle x(t), y(t) \rangle$

$t = a$ to b

parametric eqs of spiral

$$\int_a^b \vec{F}(x(t), y(t))\sqrt{x'^2 + y'^2}\,dt$$

Q) is $\vec{F}$ conservative?

$$\vec{F} = \langle y + \cos x, x \rangle$$

① check: $\frac{\partial}{\partial y}[y + \cos x] = 1$, $\frac{\partial}{\partial x}[x] = 1$ } equal so conserv.

② find f such that $\nabla f = \vec{F}$

$$\langle f_x, f_y \rangle = \langle y + \cos x, x \rangle$$

a) $\frac{\partial}{\partial x} f = y + \cos x$

b) $\frac{\partial}{\partial y} f = x \rightarrow \int df = \int x \, dy$

$$f = xy + \boxed{C(x)}$$

a) $\frac{\partial}{\partial x}[xy + C(x)] = y + \cos x$

$$y + C'(x) = y + \cos x$$

$$C' = \cos x$$

$$C = \int \cos x \, dx = \sin x + C$$

$$f = xy + \sin x + C$$

$$\int_C \vec{F} \cdot d\vec{r} = f(0,2) - f(0,0)$$

$$= 0 - 0 = \boxed{0}$$