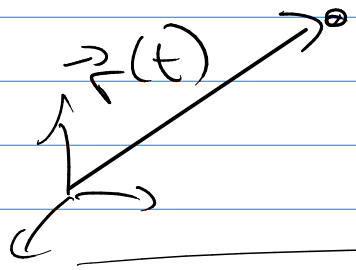
Math 344

Calc 1/2 $D(f(x))$, $I(f(x))$ ←

$\frac{d}{dx}[f(x)]$, $\int f(x)\,dx$

Ch 10 Vectors Geo. & Space

$f(t) = \langle x(t), y(t), z(t) \rangle$



Ch 11 $f(x,y)$, $f(x,y,z)$

(change) | partial derivatives

Ch 12 $f(x,y)$, $f(x,y,z)$

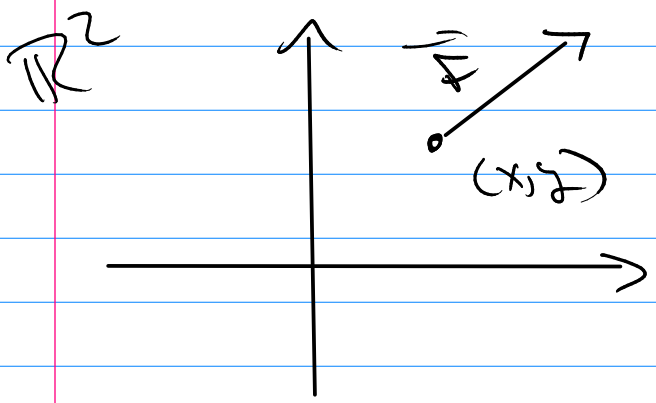
Info. Sum → Multiple Integrals

Ch 13 Vector Calculus

$f: \mathbb{R}^n \to \langle y_1, y_2, \ldots, y_n \rangle$

Vector Field

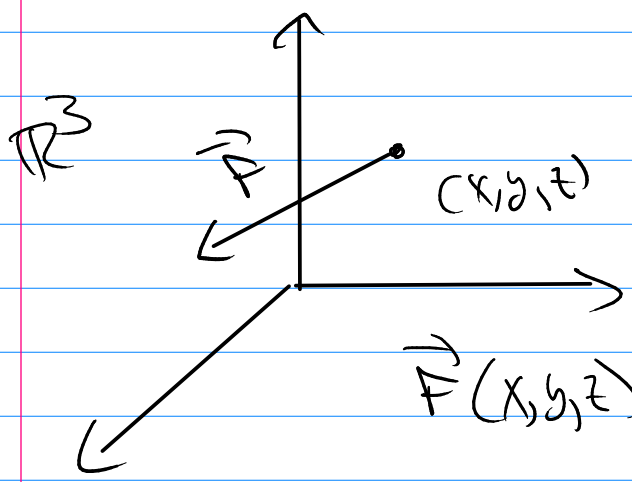
$\mathbb{R}^n$: Scalars



$\vec{F}: (x,y) \to \langle \ , \ \rangle$

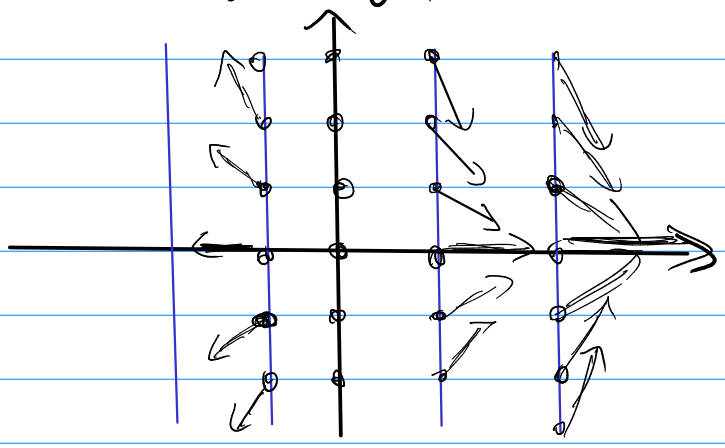
$\mathbb{R}^3$

$\vec{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$

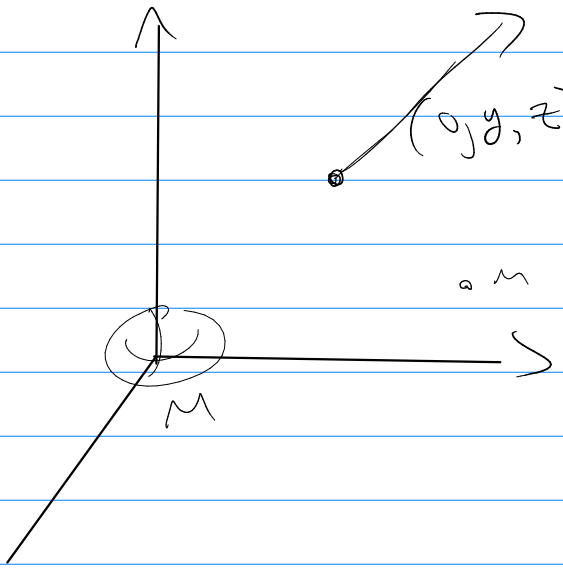


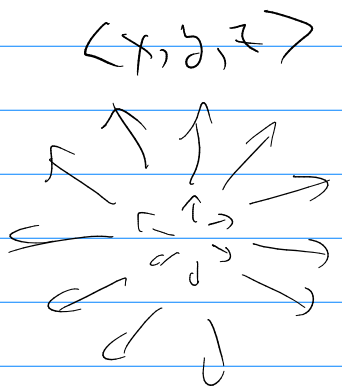
$\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$

Plot $\vec{F} = \langle x, -x \cdot y \rangle$



Ex $\vec{F}(x,y,z) = \left\langle \frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}} \right\rangle$





Ex $\vec{F}(x,y,z) = -mMG\left\langle \frac{x}{(x^2+y^2+z^2)^{3/2}}, \frac{y}{(x^2+y^2+z^2)^{3/2}}, \frac{z}{(x^2+y^2+z^2)^{3/2}} \right\rangle$

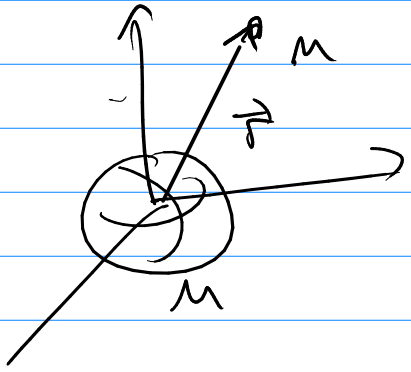
$\vec{r} = \langle x,y,z \rangle$

$\sqrt{x^2+y^2+z^2} = |\vec{r}|$

$\vec{F}(\vec{r}) = -mMG\left\langle \frac{x}{|\vec{r}|^3}, \frac{y}{|\vec{r}|^3}, \frac{z}{|\vec{r}|^3} \right\rangle$

$\vec{F}(\vec{r}) = -\frac{mMG}{|\vec{r}|^3}\vec{r}$

Vector field (Gravity)



$$\nabla f(x,y,z) = \langle f_x, f_y, f_z \rangle$$

$$= \langle f_x(x,y,z),\ f_y(x,y,z),\ f_z(x,y,z) \rangle$$

So the gradient <u>was</u> a <u>vector field</u>.

Note: given $\vec{F} \overset{?}{=} \nabla f$

if $f(x,y,z)$ (scalar function) exists such that $\nabla f = \vec{F}$

we call $\vec{F}$ a <u>conservative vector field</u>

→ we call f the <u>potential function</u> of $\vec{F}$

Ex $f(x,y,z) = \dfrac{mMG}{\sqrt{x^2+y^2+z^2}}$

For home ∇f ?