Math 344

Applications:

① Volume $\quad V = \iint_R f \, dA$

② Area of R $\quad A = \iint_R (1) \, dA$

③

← flat sheet (lamina)

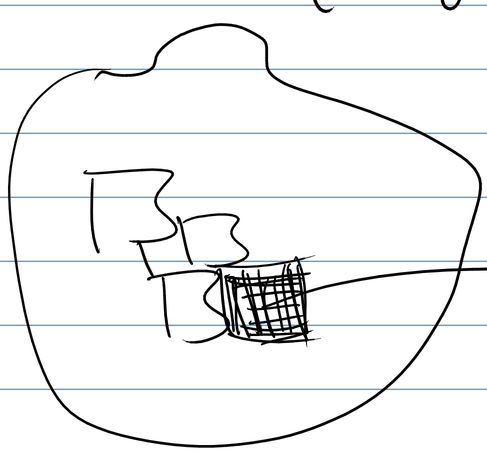
$\rho(x,y) = \frac{\text{mass}}{\text{area}}$

density function.

density function: $\rho(x,y) = \frac{\text{mass}}{\text{area}}$

density function. $\sigma(x,y) = \frac{\text{charge}}{\text{area}}$

$\rho(x,y)\, dA \rightarrow \frac{\text{mass}}{\cancel{\text{area}}} \cdot \cancel{\text{area}} \rightarrow \text{Mass}_i$

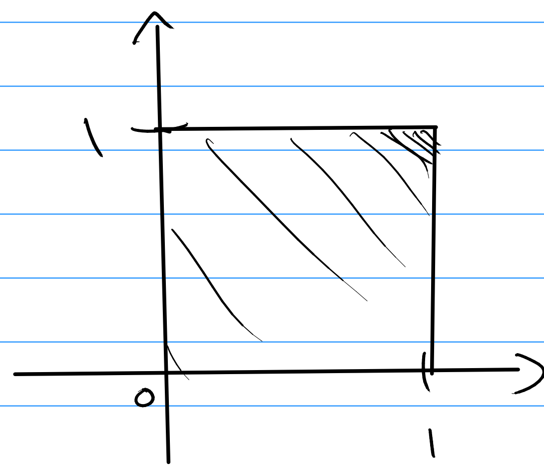


$\rho \, dA = \text{Mass}_i$

$\iint_R \text{Mass}_i = \boxed{\text{total Mass}}$

(3) Mass of lamina with density function $\rho(x,y)$ $= \boxed{\iint_R \rho(x,y)\, dA}$

ex



$\rho(x,y) = x+y$

$M = \iint_R (x+y)\, dA$

$M = \int_0^1 \left(\int_0^1 (x+y)\, dx \right) dy = \underline{\text{you finish!}}$

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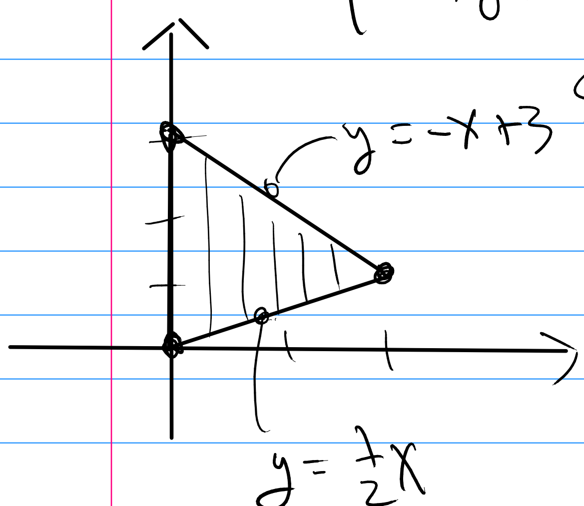
ex R is the triangle w/ vertices $(0,0), (2,1), (0,3)$

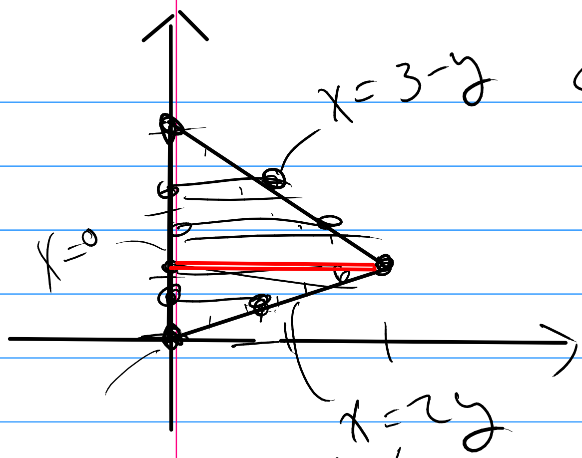
$\rho(x,y) = 1 + x^2 + y^2$



Cartesian!

$\iint_R (1+x^2+y^2)\, dA$

$M = \int_0^2 \left(\int_{\frac{1}{2}x}^{3-x} (1+x^2+y^2)\, dy \right) dx$



Cartesian:

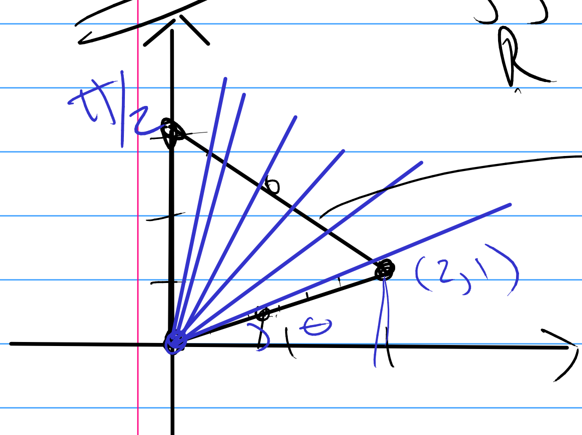
$$\iint_R (1+x^2+y^2)\,dA$$

$$= \int \left(\int (1+x^2+y^2)\,dx \right) dy$$

$$M = \int_0^1 \left(\int_0^{2y} (1+x^2+y^2)\,dx \right) dy + \int_1^3 \int_0^{3-y} (1+x^2+y^2)\,dx\,dy$$

Polar?

$$\iint_R (1+x^2+y^2)\,dA$$



$y = 3-x \qquad x+y=3 \qquad r\cos\theta + r\sin\theta = 3$

$$r = \frac{3}{\cos\theta+\sin\theta}$$

$$\tan\theta = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\tan^{-1}\left(\frac{1}{2}\right) \le \theta \le \pi/2 \qquad 0 \le r \le \frac{3}{\cos\theta+\sin\theta}$$

$$M = \int_{\tan^{-1}(1/2)}^{\pi/2} \int_0^{3/(\cos\theta+\sin\theta)} (1+r^2)\, r\,dr\,d\theta$$

$$M = \int_0^2 \left(\int_{\frac{1}{2}x}^{3-x} (1+x^2+y^2)\, dy \right) dx$$

Inner Integral $\displaystyle\int_{1/2 x}^{3-x} (1+x^2+y^2)\, dy$

$$= y + x^2 y + \frac{1}{3} y^3 \Big|_{y=1/2 x}^{y=3-x}$$

$$= \left[(3-x) + x^2(3-x) + \frac{1}{3} \cdot \boxed{(3-x)^3} \right]$$

$$- \left[\left(\tfrac{1}{2}x\right) + x^2\left(\tfrac{1}{2}x\right) + \frac{1}{3}\left(\tfrac{1}{2}x\right)^3 \right]$$

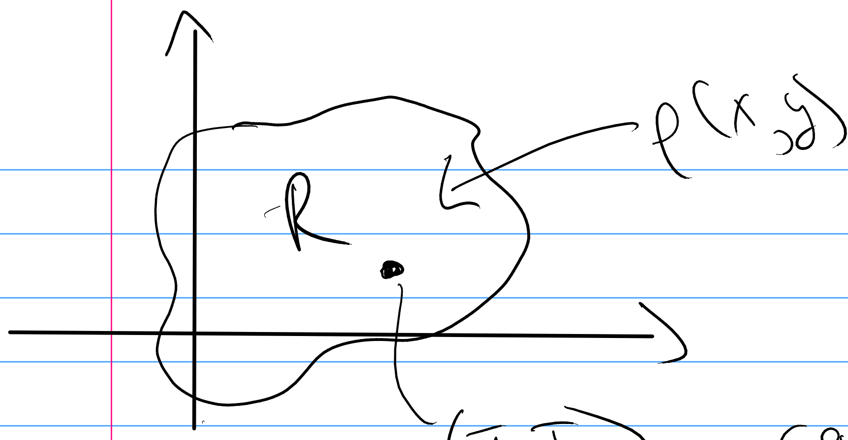
$$= \left[3 - x + 3x^2 - x^3 + 9 - 9x + 3x^2 - \tfrac{1}{3}x^3 \right]$$

$$- \left\{ \tfrac{1}{2}x + \tfrac{1}{2}x^3 + \tfrac{1}{24}x^3 \right\}$$

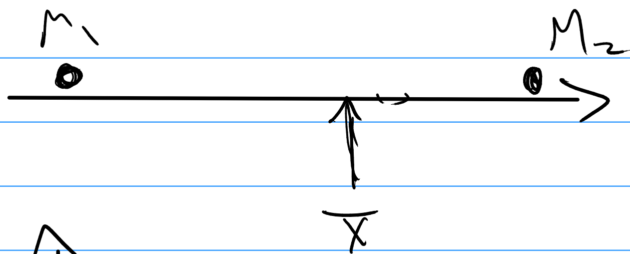
$= $ (finish)

$\circledast$

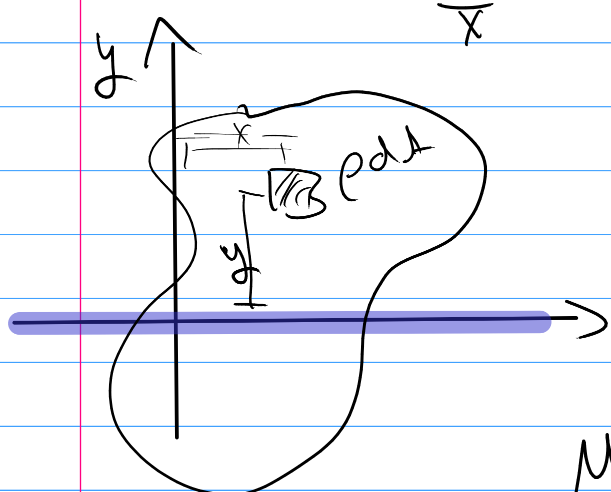
$$\Rightarrow M = \int_0^2 \circledast \, dx$$



$(\bar{x}, \bar{y})$ — center of mass



Moment = mass · displacement



$M_{\text{away from x-axis}} = M_{x\text{-axis}}$

$$M_{x\text{-axis}} = \iint_R \underbrace{y \cdot \rho\, dA}_{\text{small moments from x-axis}}$$

So

$$M_{y\text{-axis}} = \iint_R \underbrace{x \cdot \rho\, dA}_{\text{small moments from y-axis}}$$

So **Moments:** $M_x = \iint_R y \, \rho \, dA$

$$M_y = \iint_R x \, \rho \, dA$$

Center & Mass

$$\boxed{\begin{aligned} \bar{x} &= \frac{1}{m} \iint_R x \, \rho \, dA = \frac{1}{m} M_{y\text{-axis}} \\ \bar{y} &= \frac{1}{m} \iint_R y \, \rho \, dA = \frac{1}{m} M_{x\text{-axis}} \\ m &= \iint_R \rho \, dA \end{aligned}}$$