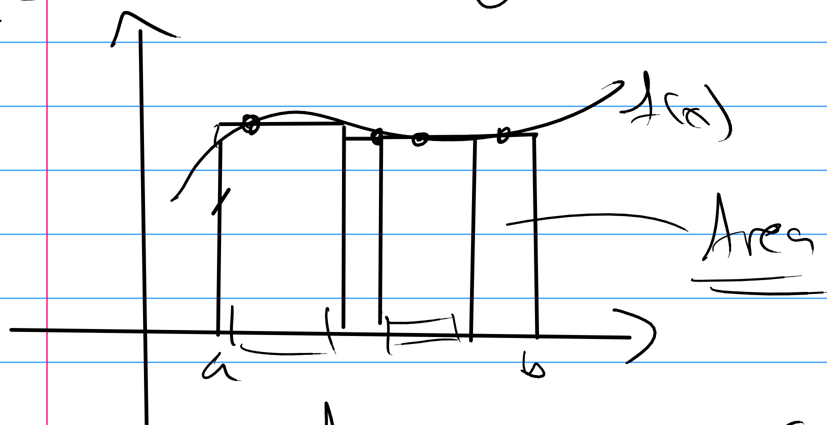
Math 399

$z = f(x,y)$ $\qquad$ $w = f(x,y,z)$

Functions of many ind. variables

Ch 12 Integration (Calc 2 → Definite Integral)

Area



Area $\approx$ Sum of known rectangles

$$\text{Area} = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} \underbrace{\underbrace{f(x_i^*)}_{\text{height}} \underbrace{\Delta x_i}_{\text{width}}}_{\text{area of } A_i}$$

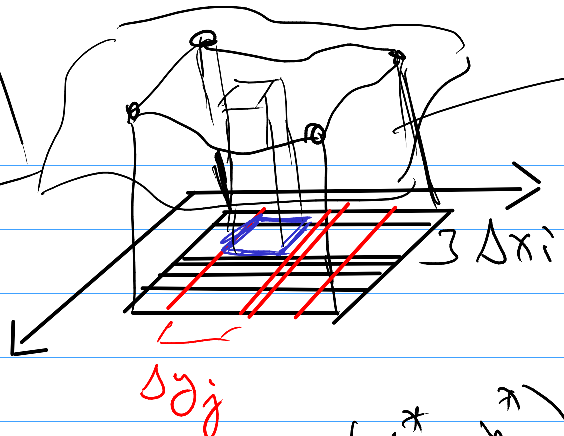
Part of Fund. thm of Calc

$$\boxed{\int_a^b f(x)\,dx} = \left[\int f(x)\,dx\right]\Big|_a^b = F(b) - F(a)$$

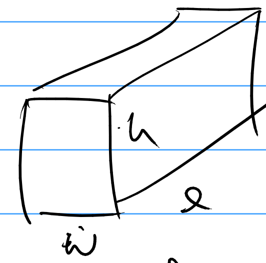
You can approx with

left endpt, right endpt, midpt, trap, Simp rules

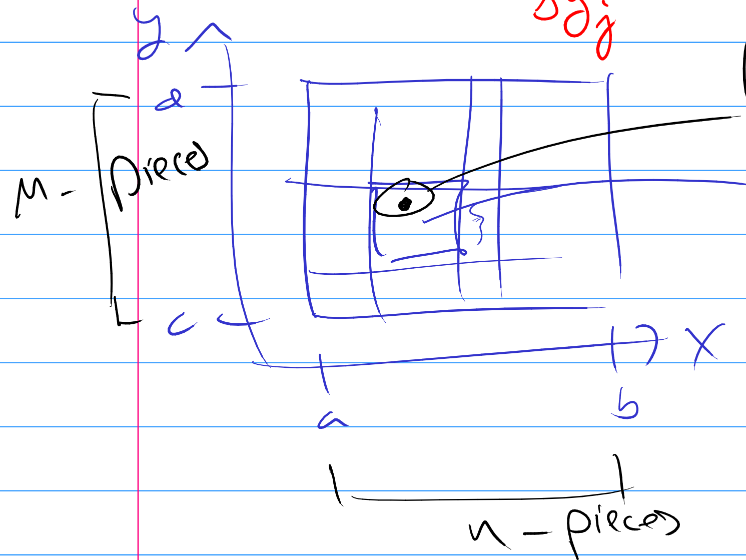
$\boxed{z = f(x,y)}$



Volume?



$V = l \cdot w \cdot h$



(x_{ij}^*, y_{ij}^*)

ΔA_{ij}

$\text{height} = f(x_{ij}^*, y_{ij}^*)$

$$V_{ij} = f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij}$$

$$V \approx \sum_{j=1}^{m} \sum_{i=1}^{n} V_{ij}$$

$$V = \lim_{\Delta x_i, \Delta y_j \to 0} \sum_{j=1}^{m} \sum_{i=1}^{n} V_{ij}$$

$$\boxed{V = \lim_{\Delta x_i, \Delta y_j \to 0} \sum_{j=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij}}$$

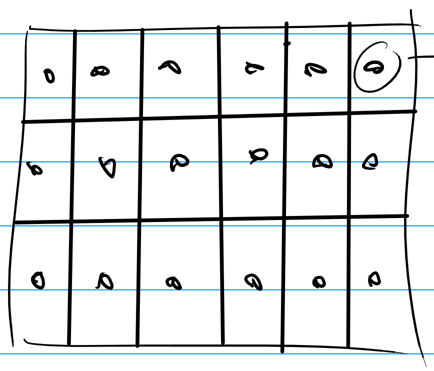
$$V = \iint_{R = [a,b] \times [c,d]} f(x,y)\,dA$$

Regular Double Rieman Sum

$\Delta x = \frac{b-a}{n}$ $\quad \Delta y = \frac{d-c}{m}$ $\quad \Delta A = \Delta x \Delta y$

$$V = \lim_{n\to\infty}\lim_{m\to\infty}\sum_{j=1}^{m}\sum_{i=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta x\,\Delta y$$

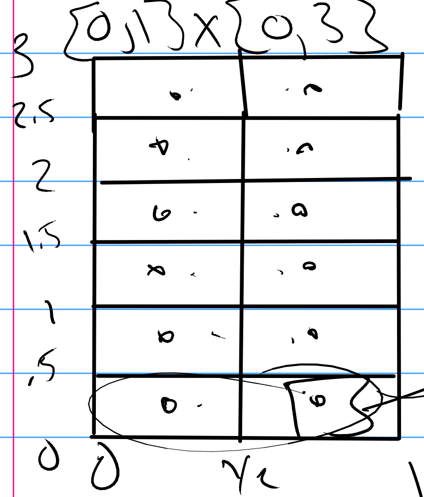
Mid pt: $x_{ij}^* = \frac{x_{i-1}+x_i}{2}$ $\quad y_{ij}^* = \frac{y_{j-1}+y_j}{2}$



(x_{ij}, y_{ij})

Approx means $n, m \not\to \infty$ $\quad n$ = fixed number
m = fixed number

ex $\iint\limits_{[0,1]\times[0,3]} (x^2+\sin(y))\,dA$ $\quad$ x's use $n=2$
y's use $m=6$



midpts $(.25, .25)$ $\quad (.75, .25)$

$\Delta A = \left(\frac{1-0}{2}\right)\left(\frac{3-0}{6}\right) = 1/4 = .25$

$V_1 = (.75^2 + \sin(.75))(.25)$

Or Use Iterated Integrals

$\boxed{\text{Fubini's thm}}$

x's y's

$f(x,y)$ is cont. on $R: [a,b] \times [c,d]$

$$V = \iint_R f(x,y)\,dA = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

$$= \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy$$

Special Case: $f(x,y) = h(x) \cdot g(y)$

$$V = \left[\int_a^b h(x)\,dx \right] \left[\int_c^d g(y)\,dy \right]$$

ex $$\iint_{[0,1]\times[0,3]} (x^2 + \sin(y))\,dA = \int_0^3 \left(\underbrace{\int_0^1 (x^2 + \sin y)\,dx} \right) dy$$

$$= \int_0^3 \left[\tfrac{1}{3}x^3 + x \sin y \right]_{x=0}^{x=1} dy$$

$$= \int_0^3 \tfrac{1}{3} + \sin y \; dy = \tfrac{1}{3}y - \cos y \Big]_0^3$$

$$= (1 - \cos 3) - (0 - 1)$$

$$= \boxed{2 - \cos 3}$$