Math 344

Rel. Extrema: $z = f(x,y)$

(1) **Critical's** a) $\left\{ \begin{array}{l} f_x = 0 \\ f_y = 0 \end{array} \right]$ Solve system of eq's

(Horiz. tangents)

b) f_x dne or f_y dne

the Critical's are possible locations of rel. extrema.

(2) 2nd Derivatives test (only to horiz. tang)

$$D = f_{xx} f_{yy} - (f_{xy})^2$$

a) $D < 0$ Saddle

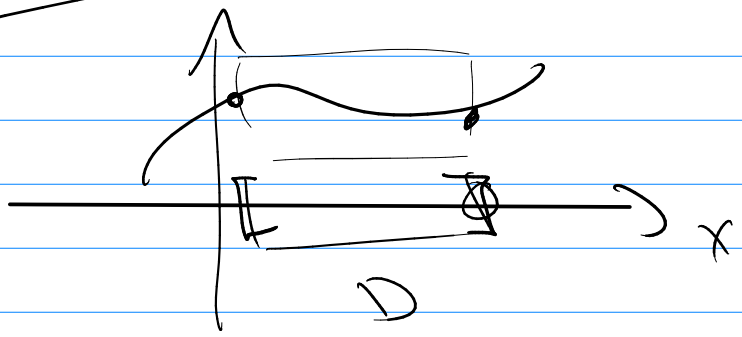
b) $D > 0 \rightarrow f_{xx} < 0$ rel. max location

$\rightarrow f_{xx} > 0$ rel. min location

c) $D = 0$ test fails

Abs. Extrema on a closed set D

calc 1

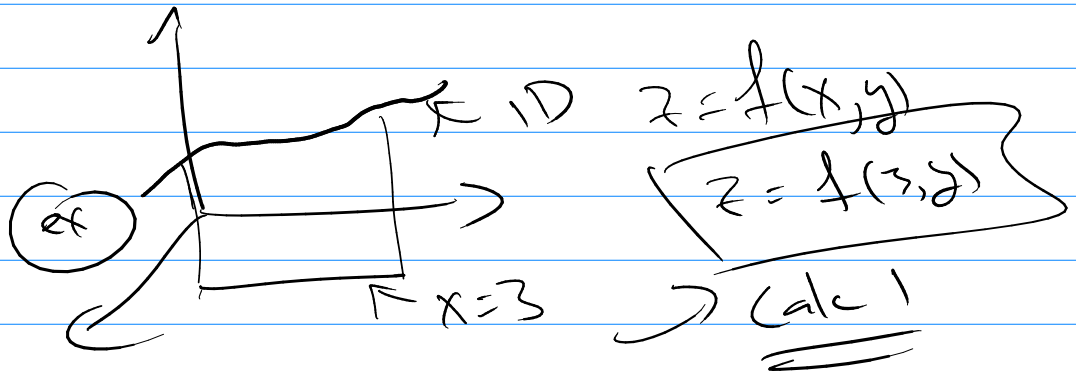


← we will have abs max and abs min.

Where? rel. extrema or on D's boundry

How to find it?

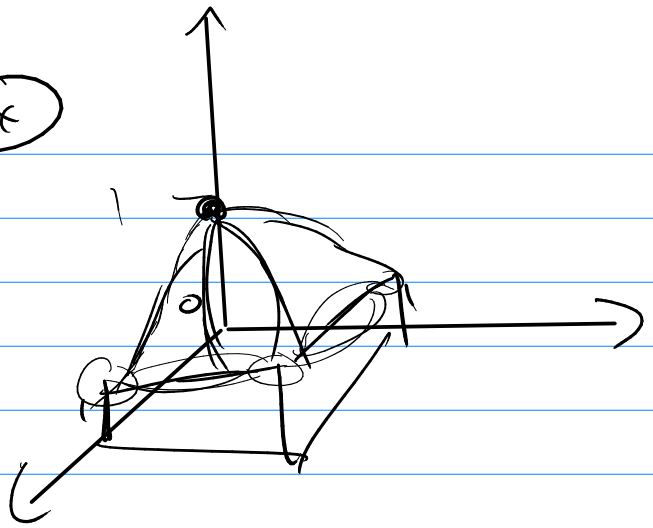
1. Critical's
2. abs. extrema on boundry



3. largest $f(\,) = \max$

   smallest $f(\,) = \min$

Ex



$z = \cos(x^2+y^2)$

① Critical:

$f_x = -2x \sin(x^2+y^2)$

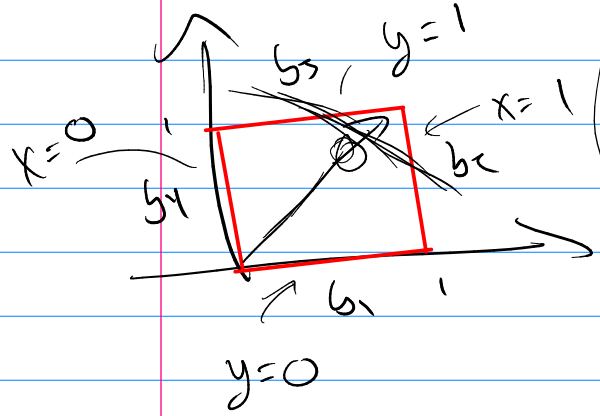
$f_y = -2y \sin(x^2+y^2)$

Solve: $-2x\sin(x^2+y^2) = 0$ (on $[0,1]\times[0,1]$)

$-2y\sin(x^2+y^2) = 0$

$(0,0)$

② Boundary: $z = \cos(x^2+y^2)$



b_1 $z = \cos(x^2)$ on $[0,1]$

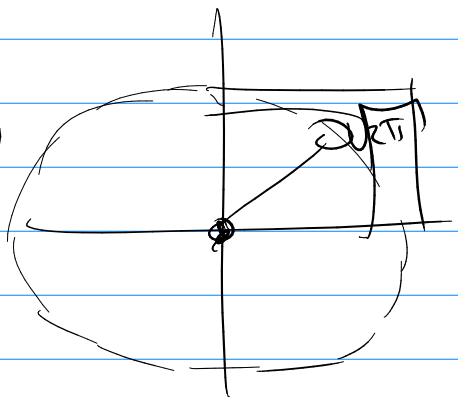
b_2 $z = \cos(1+y^2)$ on $[0,1]$

b_3 $z = \cos(1+x^2)$ on $[0,1]$

b_4 $z = \cos(y^2)$ on $[0,1]$

$\sin(x^2+y^2) = 0$

$x^2+y^2 = (\sqrt{2}\pi)^2$



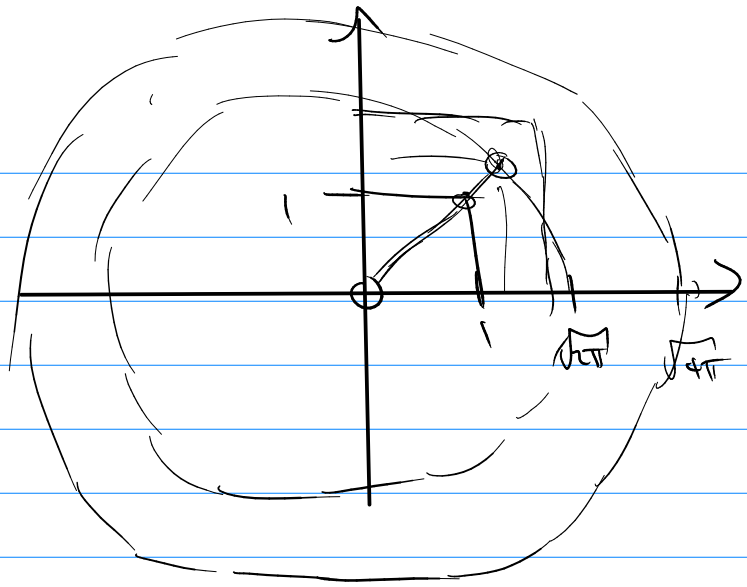
Solve $\sin(x^2+y^2)=0$

$$x^2+y^2=2n\pi$$

$$x^2+y^2=2\pi$$

$$r=\sqrt{2\pi}$$

$$\sqrt{6\pi}>\sqrt{2}$$

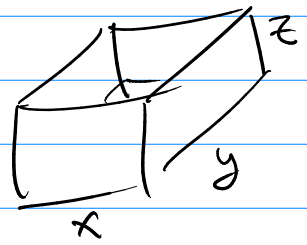


Note: A lot of max/min probs have...

$$w=f(x,y,z)$$

→ with respect to. $g(x,y,z)=K$

ex $V=xyz$



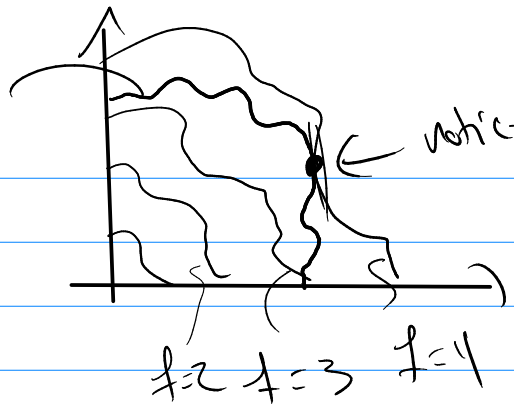
Max: Given you only have $10\ ft^2$ of material.

$$f(x,y,z)=xyz$$

$g(x,y,z)=K$ ∴ $2xz+2yz+xy=10$

$\mathbb{R}^2$

$g = k$



← notice extreme @ gradient ∥ parallel to gradient g

Finding Min/Max with constraint ...

(Lagrange's Method)

① Solve system of eqn's

$$\nabla f = \lambda \nabla g$$

$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ f_z = \lambda g_z \\ g(x,y,z) = k \end{cases}$$

4 eqn's and 4 unknowns.

② Find all x, y, z, λ solns.

→ $f(x,y,z)$ = largest? ← Max

→ $f(x,y,z)$ = smallest? ← Min

Two constraints.

Max/Min $\omega = f(x,y,z)$

with respect to $g(x,y,z) = K$ $\quad h(x,y,z) = C$

Solve system of eqns

$$\begin{cases} \nabla f = \lambda \nabla g + \mu \nabla h \\ g = K \\ h = C \end{cases}$$

↳ Find all (x,y,z), λ, μ