Math 344

Exam 1 What to do Sat/Sun?

① Know Def., Deriv's!

② Know Integration Tech's!

about 10 probs. | from 8 sections (+) | end of 10

→ 1 extra credit

11.7 $z = f(x,y)$

Def: (a,b) has a local extrema (local max / local min) of $f(a,b)$ if:

$f(x,y) \le f(a,b)$ near (a,b) → local max @ (a,b) of $f(a,b)$

$f(x,y) \ge f(a,b)$ near (a,b) → local min @ (a,b) of $f(a,b)$

Thm If f has a local max/min @ (a,b) and $f_x(a,b)$, $f_y(a,b)$ exist

$\rightarrow f_x(a,b)=0$, $f_y(a,b)=0$

(basically says we have horz. tangent plane)

possible locations of local extrema are...

horz. tangents ① $f_x=0$ and $f_y=0$

edges ② f_x dne or f_y dne

] Critical points

ex Find Critical points:

$$z=f(x,y)=x^2-y^2$$

Criticals $f_x=2x$ $f_y=-2y$

① d.n.e.? No.

② (horz. tangents) $2x=0$ $-2y=0$

$(0,0)$

extrema here?

$f(x,y) = x^2 - y^2$

Saddle point.

let $y = 0$ $\quad z = x^2$

let $x = 0$ $\quad z = -y^2$

$f(x,y) \leq 0$?

Not $f(x,y) \geq 0$?

No extrema.

Th^n 2nd Derivatives Test

① 2nd Partials $(f_{xx}, f_{xy}, f_{yx}, f_{yy})$ are cont about (a,b)

$f_x(a,b) = 0$, $f_y(a,b) = 0$

② let $D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2$

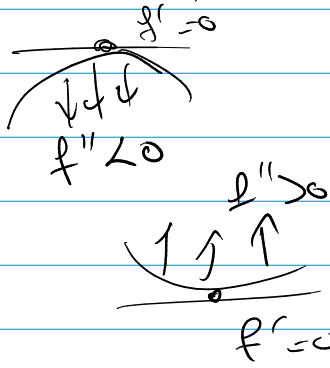
a) $D < 0$ Saddle Point (no extrema)

b) $D > 0$ $f_{xx}(a,b) > 0 \rightarrow$ local min

c) $D > 0$ $f_{xx}(a,b) < 0 \rightarrow$ local max

d) $D = 0$ test fails.

2nd Deriv.



Note: $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$

$f \to f_x \to f_{xx} \quad f_{xy}$

$f \to f_y \to f_{yx} \quad f_{yy}$

when zero?

(ex) $f(x,y) = x^2 - y^2 + 1$

$f \to f_x = 2x \to f_{xx} = 2 \quad f_{xy} = 0$

$f \to f_y = -2y \to f_{yx} = 0 \quad f_{yy} = -2$

zeros? $(0,0)$

$D = (-4) - (0) = -4 < 0$

Saddle pts

(ex) $f \to f_x = 0 \quad f_y = 0$

$f_{xx} \quad f_{xy} \quad f_{yx} \quad f_{yy}$

zeros $(1,2)$ $(-1,-2)$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$D = x^2 - y$

@ $(1,2)$ $D = (1)^2 - (2) = -1 < 0$

Saddle pt @ $(1,2)$

@ $(-1,-2)$ $D = (-1)^2 - (-2) = 3 > 0$

$\rightarrow$ rel max/min @ $(-1,-2)$