Math 344

11.5 Implicit Differentiation.

① Given your implicit equation

→ write it as $\underbrace{(\text{expression})}_{F(x_1, x_2, \dots, x_n)} = 0$

x_n ← dependent variable

$$\boxed{\frac{\partial x_n}{\partial x_i} = -\frac{F_{x_i}}{F_{x_n}}}$$

Calc 1 type: $\frac{dy}{dx}$ $\quad y^2 + x^2 - xy = 3\sin(y)$

Calc 1 $\quad \frac{d}{dx}\left[y^2 + x^2 - xy = 3\sin y \right]$

$$2yy' + 2x - [y + xy'] = 3\cos y \, y'$$

$$y' = \frac{y - 2x}{2y - x - 3\cos y}$$

using 11.5

$$y^2 + x^2 - xy = 3\sin(y)$$

$$F = \boxed{y^2 + x^2 - xy - 3\sin y} = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = - \frac{2x - y}{2y - x - 3\cos y}$$

why?

$$F(x,y) = 0$$

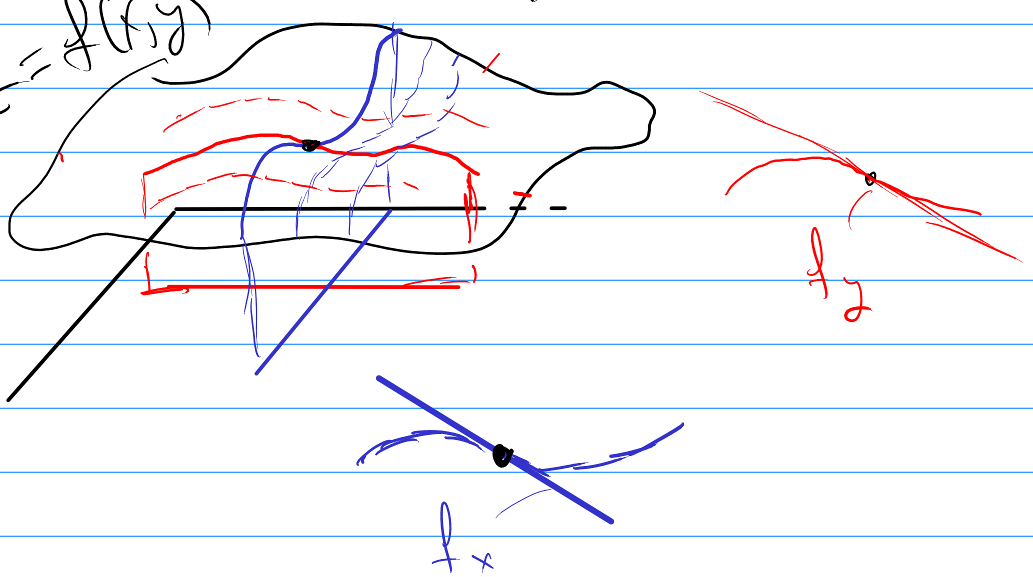
$$F_x \frac{dx}{dx} + F_y \frac{dy}{dx} = 0$$

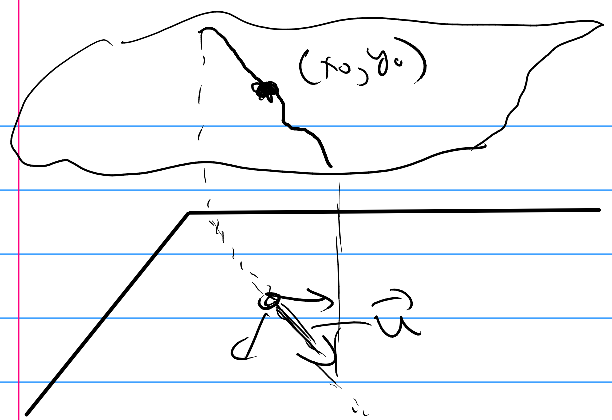
"1"

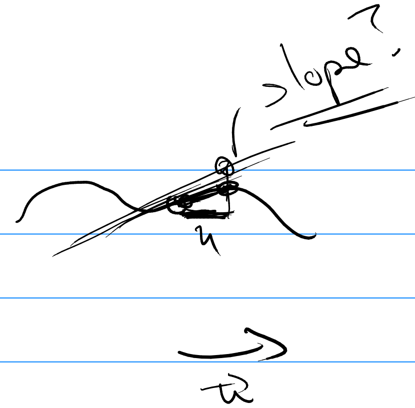
$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

11.6 Directional Derivatives / Gradient

Partials: $z = f(x,y)$





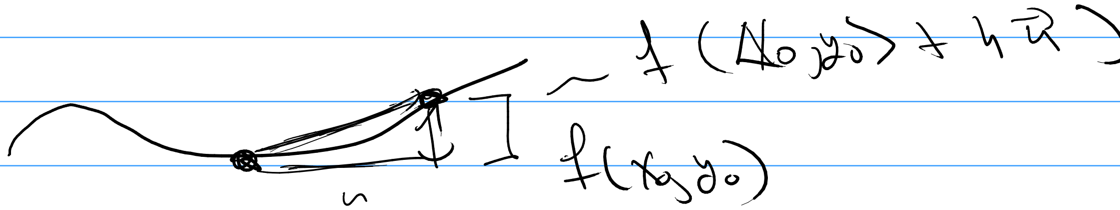


$D_{\vec{u}} f$

① $\vec{u}$ is a unit vector. $u = \langle u_1, u_2 \rangle$

② $D_{\vec{u}} f(x,y)$ to stretch $\vec{u}$ by h

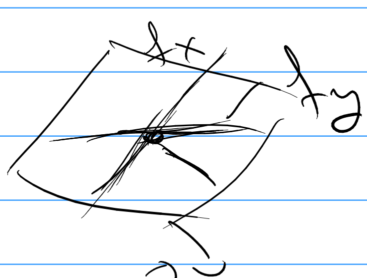
$$h\vec{u} = \langle hu_1, hu_2 \rangle$$

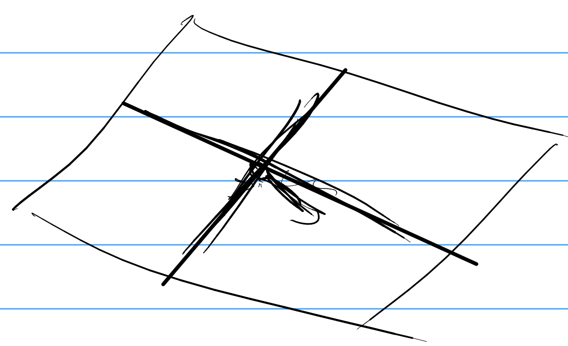


$$D_{\vec{u}} f(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0+hu_1, y_0+hu_2) - f(x_0,y_0)}{h}$$

$$= \lim_{h\to 0} \frac{f(\vec{x}_0 + h\vec{u}) - f(\vec{x}_0)}{h}$$

thm: $D_u f(x,y) = u_1 f_x + u_2 f_y$





Consider: $D_{\vec{u}} f = u_1 f_x + u_2 f_y$

$$= \underbrace{\langle f_x, f_y \rangle} \cdot \langle u_1, u_2 \rangle$$

$f(x,y,z)$

$$D_{\vec{u}} f = \underbrace{\langle f_x, f_y, f_z \rangle} \cdot \langle u_1, u_2, u_3 \rangle$$

looks like an operator to me...

$f(x,y,z)$

$\downarrow$

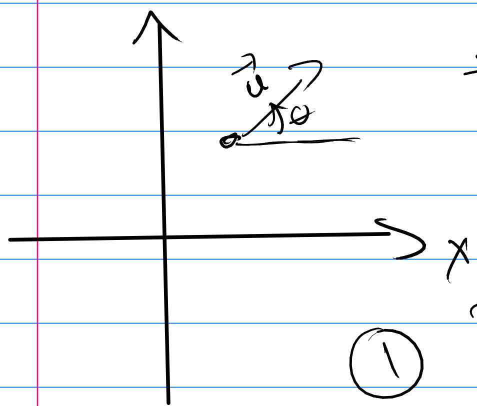
$\boxed{\text{gradient}}$ use ∇ as its symbol.

$\langle f_x, f_y, f_z \rangle$

$$\nabla f = \langle f_x, f_y, f_z \rangle$$

So $\boxed{D_{\vec{u}} f = \nabla f \cdot \vec{u}}$

What can we know b/c $D_{\vec{u}} f = \nabla f \cdot \vec{u}$



$\vec{u} = \langle \cos\theta, \sin\theta \rangle$

① $D_{\vec{u}} f = f_x \cos\theta + f_y \sin\theta$

② $|D_{\vec{u}} f| = |\nabla f \cdot \vec{u}| = |\nabla f| \, |\vec{u}|^{=1} \cos\theta$

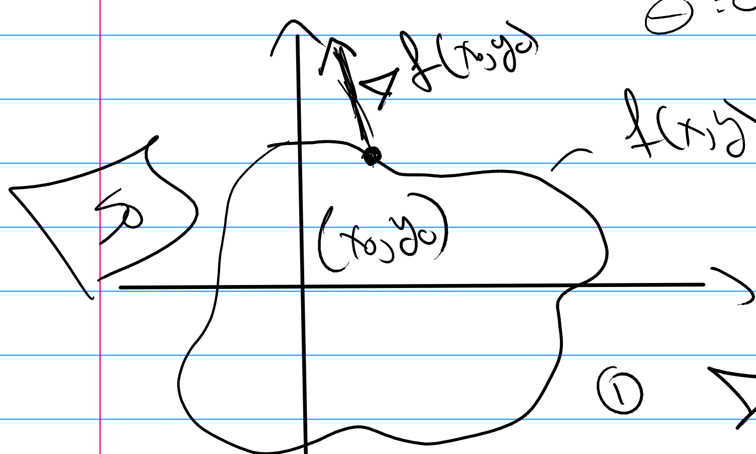
$|D_{\vec{u}} f| = |\nabla f| \cos\theta$

a) @ $\underline{\theta = 0}$ $|D_{\vec{u}} f|$ is a max

& $\boxed{|\nabla f|}$

b) Max is in $\underline{\text{direction}}$ of ∇f

$\theta = 0$



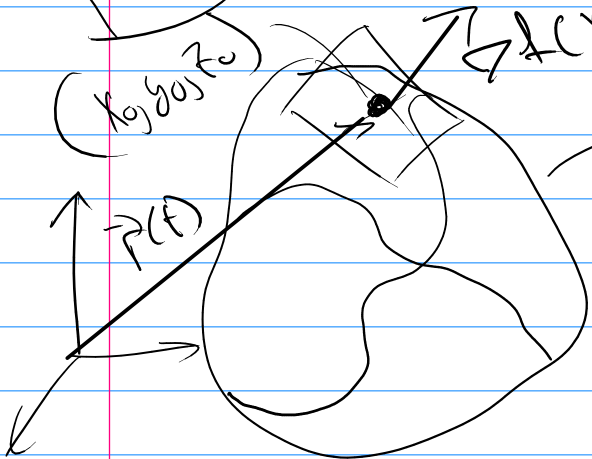
$f(x,y) = K$ (level curve)

@ (x_0, y_0)

① $\nabla f(x_0, y_0) = \langle f_x(x_0,y_0), f_y(x_0,y_0) \rangle$
is direction of steepest ascent

② $|\nabla f(x_0, y_0)| =$ steepest slope

$\mathbb{R}^3$ $\quad w = f(x,y,z)$



$f(x,y,z) = K$

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$\vec{r}(t_0) = \langle x_0, y_0, z_0 \rangle$$

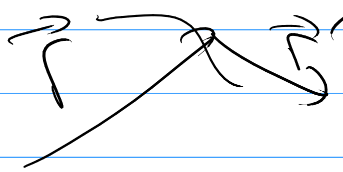
Parameterize.

$$f(x,y,z) = K$$

$$\frac{d}{dt}\left[f(x(t), y(t), z(t)) = K \right]$$

$$f_x \frac{dx}{dt} + f_y \frac{dy}{dt} + f_z \frac{dz}{dt} = 0$$

$$\nabla f \cdot \vec{r}\,' = 0$$



$$\nabla f \perp \vec{r}\,'$$

use as tang. **plane's normal.**

tangent vector.