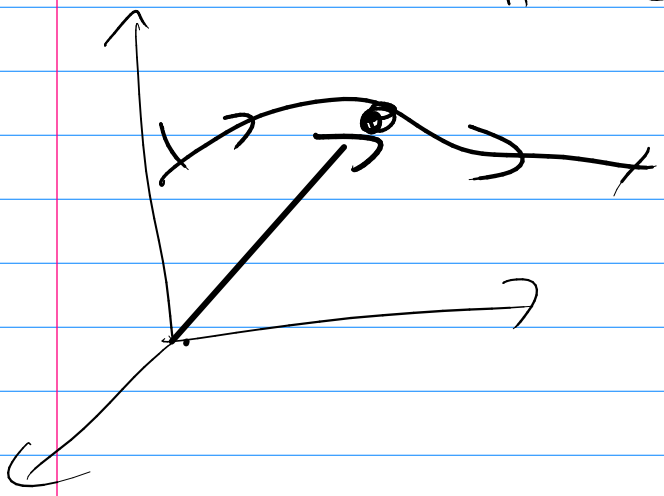
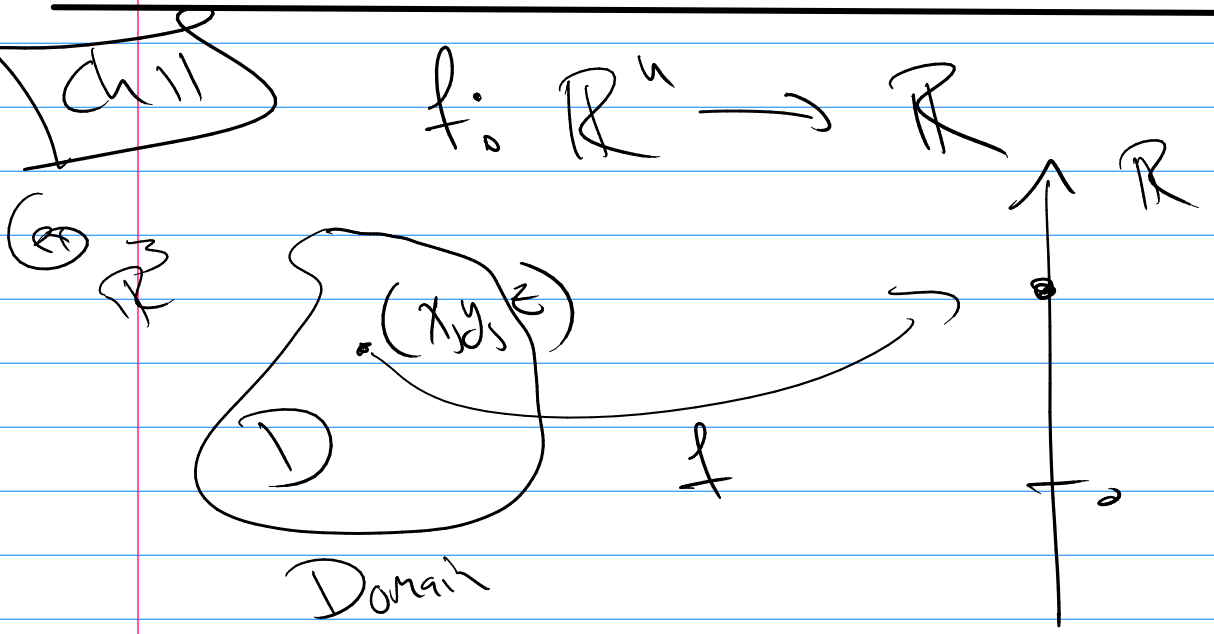
Math 399

Ch 10 $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

$\vec{r}: \mathbb{R} \to \begin{pmatrix} 3D \\ \text{vectors} \end{pmatrix}$



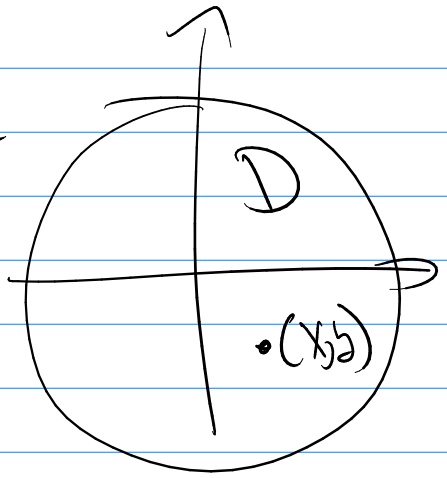
Ch 11 $f: \mathbb{R}^n \to \mathbb{R}$



$f(\underbrace{x_1, x_2, \ldots, x_n}_{n\text{-tuple}}) = \underbrace{d}_{\text{single real number (Scalar)}}$

Typically:

D on $\mathbb{R}^2$



① $f(x,y)$

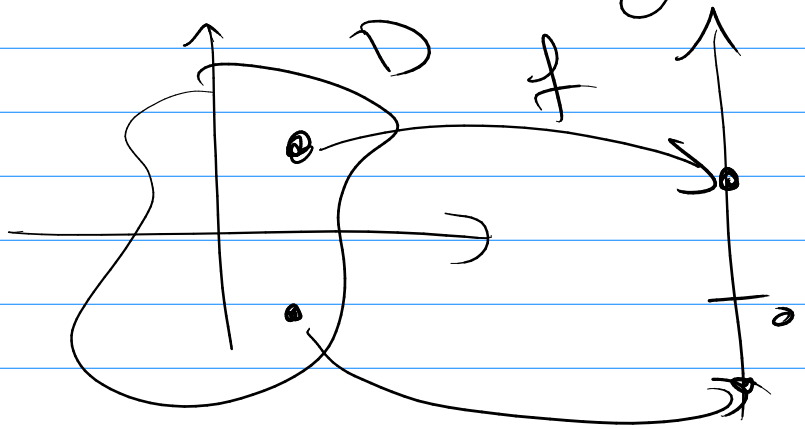
② (x,y) ordered pair

③ x,y independent variables

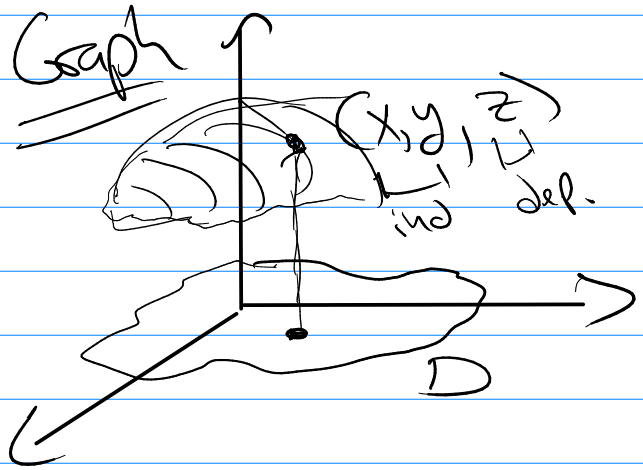
④ $z = f(x,y)$ z is the dependent variable

Visualize:

① Arrow Diagram



② Graph



set of all points that solve $z = f(x,y)$

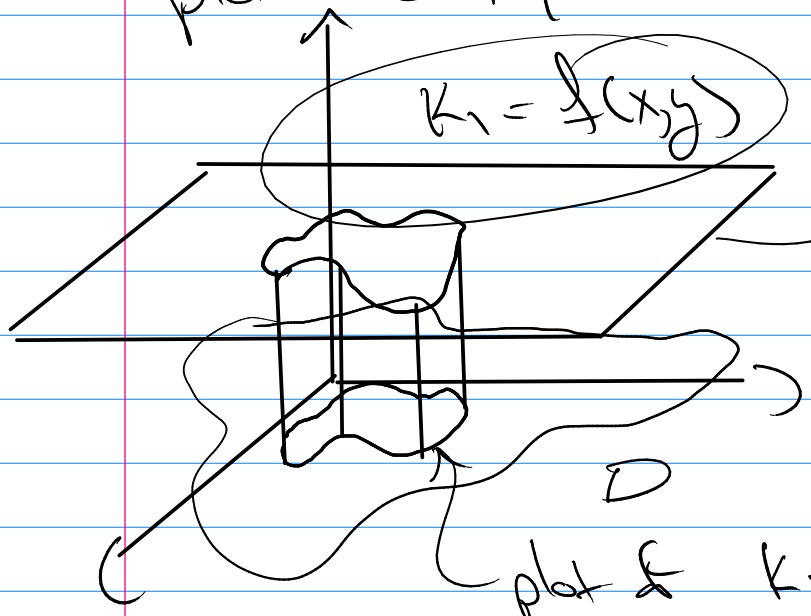
③ Contour Plot (on $\mathbb{R}^2$ you plot the @ level curves / contour curves)

$z = f(x,y)$

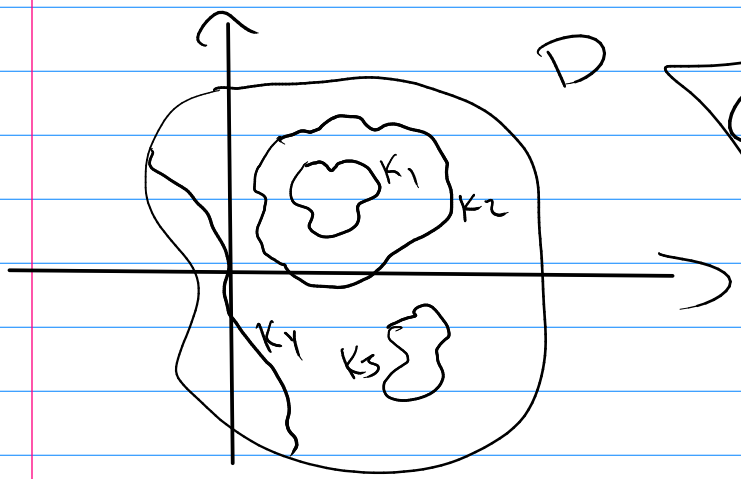
but $z = \{k_1, k_2, \ldots, k_n\}$ | Seq. of constants

④ $z = f(x,y)$

pick $z = k_1$



plot & $k_1 = f(x,y)$



D

Contour plot

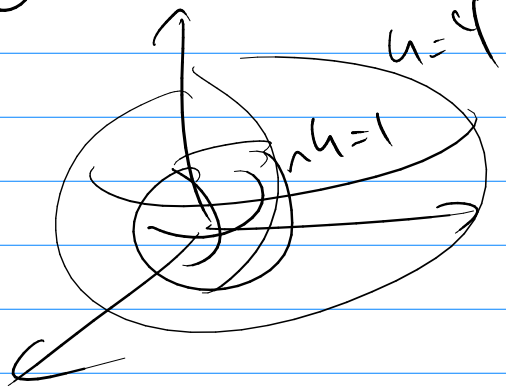
$f: \mathbb{R}^3 \to \mathbb{R}$ $\quad f(x,y,z)$

(x,y,z) ind. variables

$w = f(x,y,z)$ $\quad w$ is the dep. variable

(contour like) plot

ex $\quad w = x^2 + y^2 + z^2$



$w = 1$
$w = 4$
$w = 9$

Features/Properties of these functions?

$f: \mathbb{R}^n \to \mathbb{R}$

$d = f(x_1, x_2, \ldots, x_n)$

Calc 1/2 $\mathbb{R} \to \mathbb{R}$ $\quad y = f(x)$

$\mathbb{R}^2$ $\quad z = f(x,y)$

$\mathbb{R}^3$ $\quad w = f(x,y,z)$

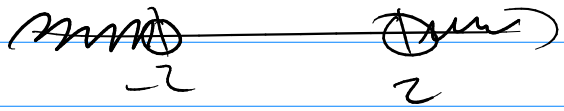
} etc

① Domain / Natural Domain

ex $y = \dfrac{1}{\sqrt{x^2-4}}$ $\quad x^2-4 \geq 0$

$(x+2)(x-2) \geq 0$

$D = (-\infty, -2) \cup (2, \infty)$



ex $z = \dfrac{1}{\sqrt{x^2-y^2}}$ $\quad (x^2-y^2) > 0$

boundry: $x^2-y^2 = 0$ (draw this dashed)

$x^2 = y^2$ $\quad y = \pm x$

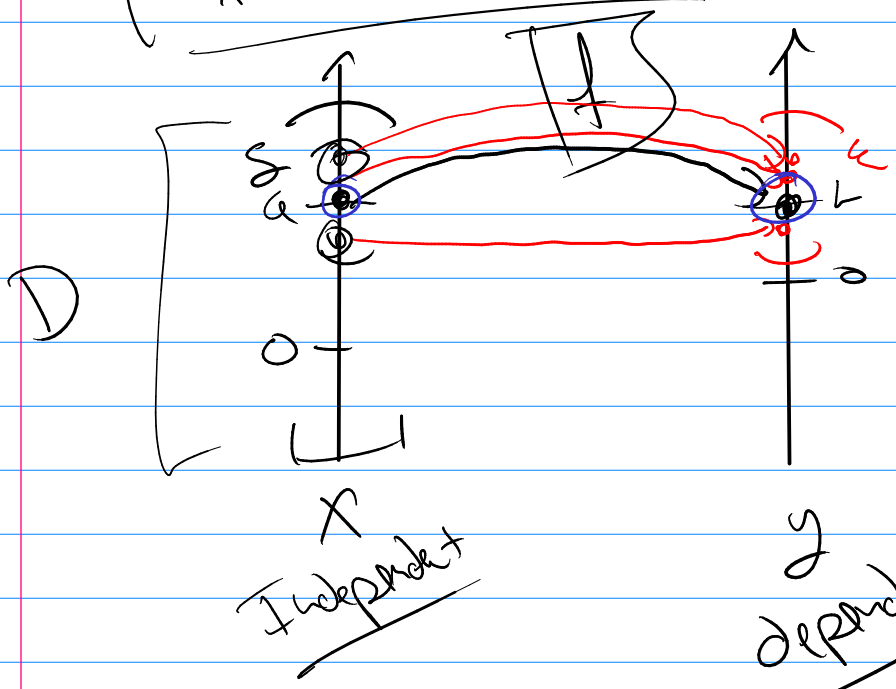
$x^2-y^2 > 0$

$z = \dfrac{1}{\sqrt{x^2-y^2}}$

Limits

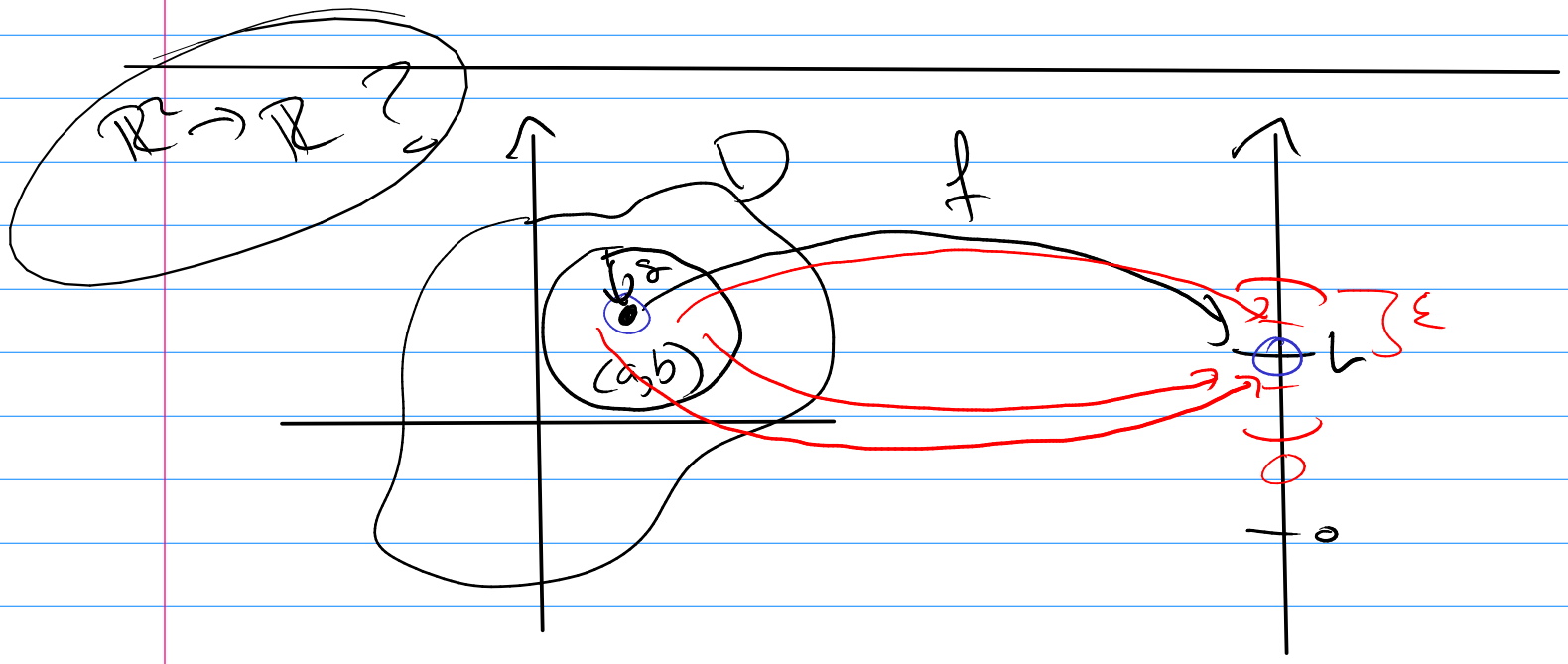
$y = f(x)$

$$\lim_{x \to a} f(x) = L$$



for all ε, there is a δ such that

$$0 < |x - a| < \delta \rightarrow |f(x) - L| < \varepsilon$$



$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

δ, ε ?