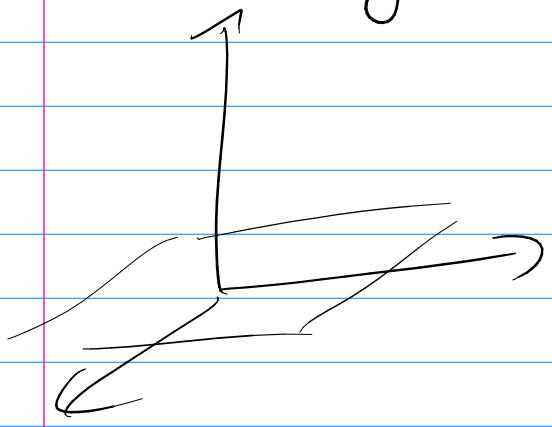
Math 344

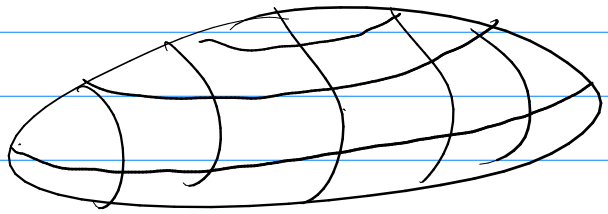
Tues: $3x^2 + y^2 + 2z^2 = 3$

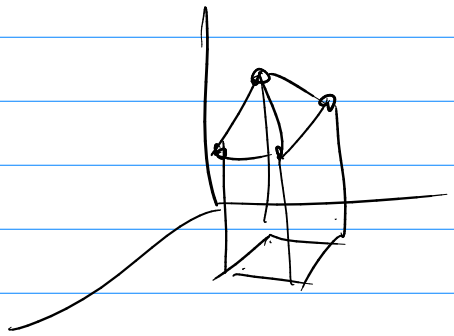


$z = 0$

$3x^2 + y^2 = 3$

$\frac{x^2}{1} + \frac{y^2}{3} = 1$



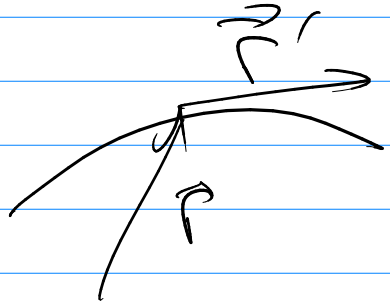


$\vec{r}(t)$ ① $\lim \vec{r}$

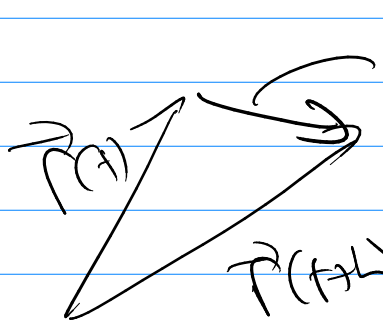
② $\frac{d}{dt}[\vec{r}]$

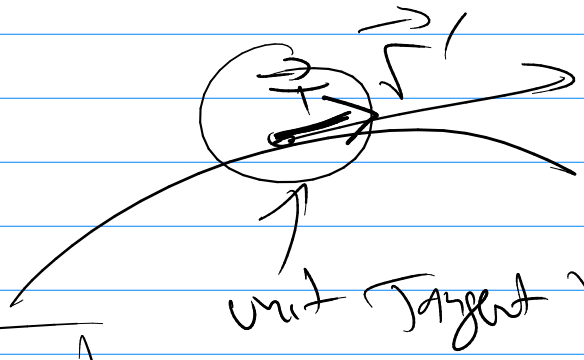
③ $\int \vec{r}\, dt$

Apps: (Derivative)



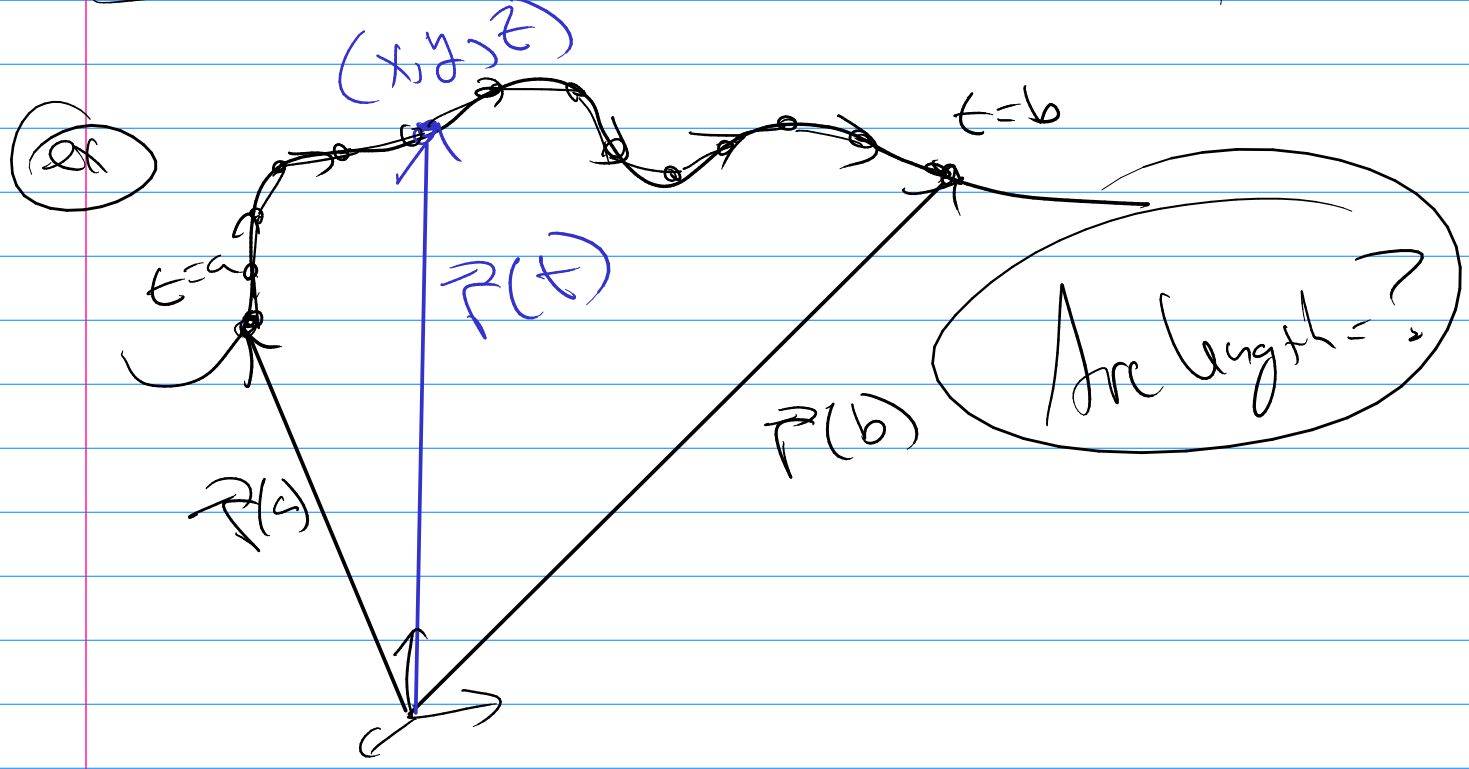
$$\vec{r}'(t) = \lim_{h \to 0} \frac{1}{h}\left(\vec{r}(t+h) - \vec{r}(t)\right)$$

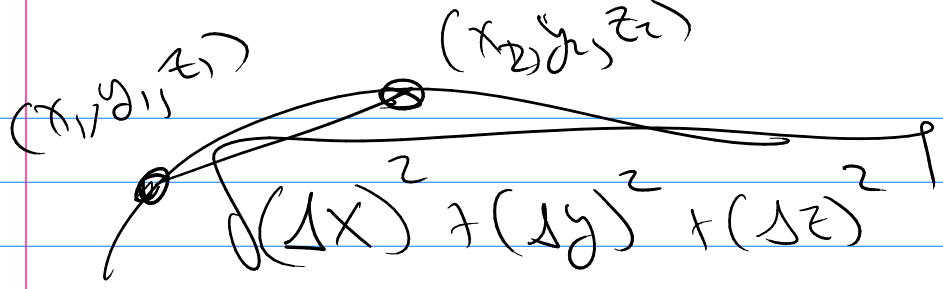




$$\vec{T} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

Applications & Integrals & $\vec{r}(t)$





$$\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

$$\int_a^b \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2 + (\Delta z_i)^2}$$

$$\Delta x \approx \frac{dx}{dt}\,dt \qquad (\Delta x^2) = \left(\frac{dx}{dt}\,dt\right)^2$$

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$

$\mathbb{R}^3$

$$L = \int_a^b \sqrt{(x')^2 + (y')^2 + (z')^2}\,dt$$

$\mathbb{R}^2$

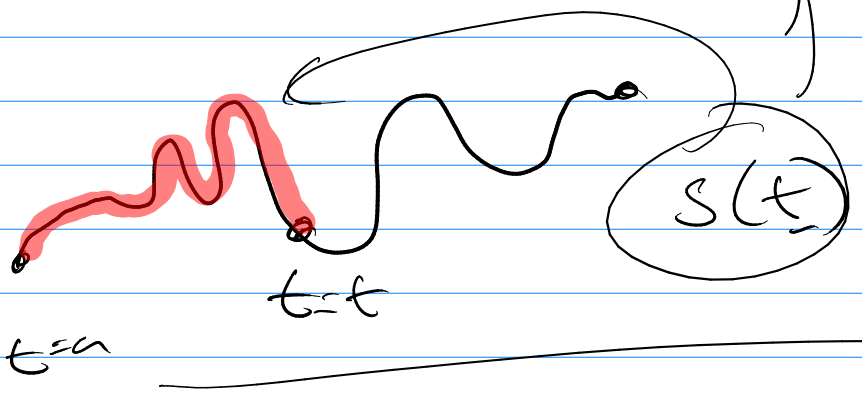
$$L = \int_a^b \sqrt{(x')^2 + (y')^2}\,dt$$

$$L = \int_a^b |\vec{r}'|\,dt$$

$\vec{r} = \langle x, y, z \rangle$

$\vec{r}' = \langle x', y', t' \rangle$

How far have I gone so far? (as a function)



$$s(t) = \int_a^t |\vec{r}'(u)|\, du$$

Also, Fund. Th^m of Integrals

$$\frac{ds(t)}{dt} = |\vec{r}'(t)|$$

If you do $s(t) = \int_a^t |r'| du$ first

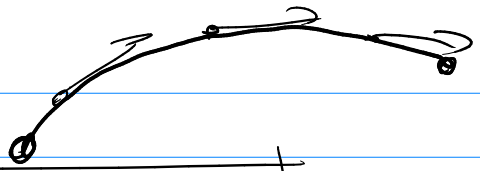
$s = s(t)$ → solve for t

$t(s)$ ← all s's

$$\vec{r}(t) = \vec{r}(t(s)) = \vec{r}(s)$$

Arc length parameterization

Curvature



$$K = \left| \frac{d\vec{T}}{ds} \right|$$

$$\vec{T} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$K = \left| \frac{d\vec{T}}{ds} \right| = \left| \frac{d\vec{T}/dt}{ds/dt} \right| = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$$

$$K(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$$

$$K(t) = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3}$$