Math 243

Q's

A mining company estimates that the marginal cost of extracting x tons of copper ore from a mine is 0.4 + 0.008x, measured in thousands of dollars per ton. Start-up costs are $100,000. What is the cost of extracting the first 50 tons of copper?

$C' = MC(x) = (0.4 + 0.008x)$

Costs = \$100,000 @ start.

Revenue = (Sales) X

Profit = Rev − Costs

$$\int_0^{50} (0.4 + .008x)\,dx = .4x + .004x^2 \Big|_0^{50}$$

$$= \left(.4(50) + .004(50)^2\right) - (0)$$

$$= 20 + 10 = 30 \ (\$1K) = \$30,000$$

Total Costs: $100,000 + 30,000 = \boxed{\$130,000}$

$C' = MC(x) = (0.4 + 0.008x) \rightarrow C(x) = .4x + .004x^2 + C$

Costs = \$100,000 @ start.

$C(0) = \$100,000$

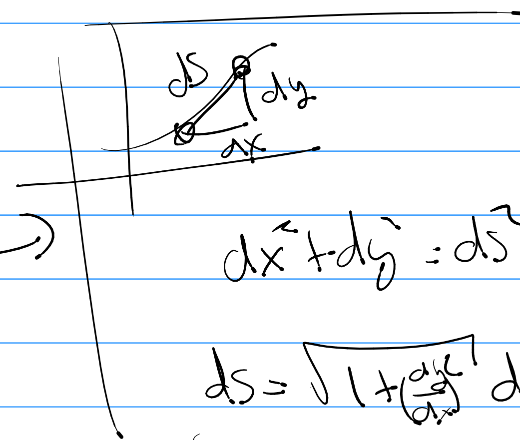
$$\boxed{C(x) = 100,000 + .4x + .004x^2}$$

$$= \frac{1}{3}x^{3/2} - x^{1/2}$$

Q3 $f(x) = \frac{1}{3}\sqrt{x}\,(x-3)$ between $x \in [16, 25]$

$$L = \int_a^b \sqrt{1 + (f')^2}\,dx$$

from



$$dx^2 + dy^2 = ds^2$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$f' = \frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$$

So $$L = \int_{16}^{25} \sqrt{1 + \left(\frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}\right)^2}\,dx$$

$$L = \int_{16}^{25} \sqrt{1 + \frac{1}{4}x - \frac{1}{2} + \frac{1}{4}x^{-1}}\,dx$$

$$L = \int_{16}^{25} \sqrt{\frac{1}{4}x + \frac{1}{2} + \frac{1}{4}x^{-1}}\,dx = \frac{1}{2}\int_{16}^{25} \sqrt{x + 2 + x^{-1}}\,dx$$

Note: $$x + 2 + x^{-1} = (\sqrt{x})^2 + 2 + \left(\frac{1}{\sqrt{x}}\right)^2$$
$$= \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$$

$$L = \frac{1}{2}\int_{16}^{25} \sqrt{\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2}\,dx = \frac{1}{2}\int_{16}^{25} \left|\sqrt{x} + \frac{1}{\sqrt{x}}\right| dx$$

$$L = \frac{1}{2}\int_{16}^{25} x^{1/2} + x^{-1/2}\,dx = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}x^{3/2} + \cancel{2}x^{1/2}\right)\Big|_{16}^{25}$$

$$L = \left(\frac{1}{3}25^{3/2} + 25^{1/2}\right) - \left(\frac{1}{3}16^{3/2} + 16^{1/2}\right)$$

$$L = \frac{125}{3} + 5 - \frac{64}{3} - 4 = \frac{61}{3} + 1 = \boxed{\frac{64}{3}}$$

Q $\int 7\sin^8 x\cos(x)\ln(\sin x)\,dx$

let $u=\sin x$ $du = \cos x\,dx$

$\int 7u^8 \ln(u)\,du = \frac{7}{9}u^9\ln(u) - \int \frac{7}{9}u^8\,du$

$$\boxed{\begin{array}{l} f(u)=\ln(u) \xrightarrow{\text{deriv}} f'(u)=\frac{1}{u} \\ g'(u)=7u^8du \xrightarrow[\text{Integrate}]{} g(u)=\frac{7}{9}u^9 \end{array}}$$

$= \frac{7}{9}u^9\ln(u) - \frac{7}{81}u^9 + C$

$= \boxed{\frac{7}{9}\sin^9 x\ln(\sin x) - \frac{7}{81}\sin^9 x + C}$

Q $\int 7e^{5x+e^{5x}}\,dx$ $a^{b+c} = a^b \cdot a^c$

$= 7\int e^{5x}\cdot e^{(e^{5x})}\,dx = \frac{7}{5}\int e^u\,du = \frac{7}{5}e^u + C$

let $u = e^{5x}$

$du = 5e^{5x}\,dx$

$= \boxed{\frac{7}{5}e^{e^{5x}} + C}$

Exam 2 Monday 17 probs @ 10pts each

→ 160 pts = 100%

7.1 – 7.8

and

8.1 – 8.4

Know ① $\int f(x)\,dx$ by recognition...

$f(x) = x^n, \frac{1}{x}, e^x, \sin(x), \cos(x),$

$\sec^2 x, \csc^2 x, \sec x \tan x, \csc x \cot x,$

$\tan x, \cot x, \sec x, \sinh x, \cosh x,$

$\frac{1}{a^2+x^2}, \frac{1}{\sqrt{a^2-x^2}}$

② do substitution..

7.1 Integration by Parts (2 probs)
7.2 Trig Integrals (2 probs)
7.3 Trig Substitution (1 prob)
7.4 Partial Fractions (1 prob)
7.5 Everything put together (2 probs)

7.6 Tables / ~~CAS~~ (2 probs)

7.7 Numeric Approximation (1 prob)

7.8 Improper Integrals (1 prob)

8.1 Arc Length (2 prob)
8.2 Surface Area (1 prob)
8.3 Hydrostatic Pressure / Force (1 prob)
8.4 Consumer Surplus / Producer Surplus (1 prob)

7.1 Integration by Parts (2 probs)

① Use it straight out...

$$\int w^p \ln(w)\,dw = \frac{1}{p+1} w^{p+1} \ln(w) - \frac{1}{p+1}\int w^p\,dw$$

let $f(w) = \ln(w) \xrightarrow{\text{deriv}} f' = \frac{1}{w}$

$g'(w) = w^p dw \xrightarrow[\text{Integrate}]{} g = \frac{1}{p+1} w^{p+1}$

$$= \boxed{\frac{1}{p+1} w^{p+1} \ln(w) - \frac{1}{(p+1)^2} w^{p+1} + C}$$

② by parts <u>after</u> a substitution

ex

$$\int 7 \sin^8 x \cos(x) \ln(\sin x)\,dx$$

let $u = \sin x$ $du = \cos x\,dx$

$$\int 7u^8 \ln(u)\,du = \frac{7}{9}u^9 \ln(u) - \int \frac{7}{9}u^8\,du$$

$f(u) = \ln(u) \xrightarrow{\text{deriv}} f'(u) = \frac{1}{u}$

$g'(u) = 7u^8 du \xrightarrow[\text{Integrate}]{} g(u) = \frac{7}{9}u^9$

$$= \frac{7}{9}u^9 \ln(u) - \frac{7}{81}u^9 + C$$

$$= \boxed{\frac{7}{9}\sin^9 x \ln(\sin x) - \frac{7}{81}\sin^9 x + C}$$

7.2 Trig Integrals (2 probs)

(1) $\sin x, \cos x$ (2) $\sec x, \tan x$

ex $\int \sin^7 x \cos^5 x \, dx$ know $\cos^2 x = (1-\sin^2 x)$

$= \int \sin^7 x \, \cos^4 x \, \cos x \, dx = \int u^7 (1-u^2)^2 \, du$

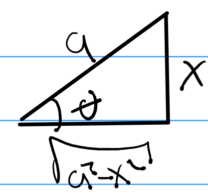
Let $u = \sin x \, dx$, $du = \cos x \, dx$ $= \int u^7 (1-2u^2+u^4) \, du$

$= \int u^7 - 2u^9 + u^{11} \, du = \frac{1}{8}u^8 - \frac{1}{5}u^{10} + \frac{1}{12}u^{12} + C$

$= \boxed{\frac{1}{8}\sin^8 x - \frac{1}{5}\sin^{10} x + \frac{1}{12}\sin^{12} x + C}$

7.3 Trig Substitution (1 prob)

you see $\sqrt{a^2-x^2}$ use $x = a\sin\theta \rightarrow$

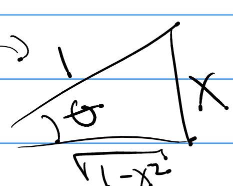


you see $\sqrt{a^2+x^2}$ use $x = a\tan\theta$

you see $\sqrt{x^2-a^2}$ use $x = a\sec\theta$

ex $\int \frac{\sqrt{1-x^2}}{x^2} \, dx = \int \frac{\sqrt{1-\sin^2\theta}}{\sin^2\theta} \cos\theta \, d\theta$

Let $x = \sin\theta \rightarrow$ $dx = \cos\theta \, d\theta$



$$= \int \frac{\cos\theta}{\sin^2\theta}\cos\theta\, d\theta = \int \frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$

$$= \int \cot^2\theta\, d\theta$$

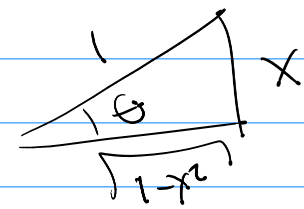
$$= \int (\csc^2\theta - 1)\, d\theta$$

$$= -\cot\theta - \theta + C$$

$$= \boxed{-\frac{\sqrt{1-x^2}}{x} - \sin^{-1}x + C}$$

$\sin^2\theta + \cos^2\theta = 1$

$\cot^2\theta = \csc^2\theta - 1$



$x = \sin\theta$

7.4 Partial Fractions

1 prob

ex $$\int \frac{1}{x^3+x}\, dx$$

$$\frac{1}{x(x^2+1)} = \frac{C_1}{x} + \frac{C_2x+C_3}{x^2+1} = \frac{1}{x} + \frac{-x}{x^2+1}$$

by numerators: $1 = C_1(x^2+1) + x(C_2x+C_3)$

$$1 = C_1x^2 + C_1 + C_2x^2 + C_3x$$

$\to x^2$: $0 = (C_1 + C_2)$

x : $0 = C_3$

const : $1 = C_1$

$C_1 = 1$

$C_2 = -1$

$C_3 = 0$

So $\int \frac{1}{x^3+x}\,dx = \int\left(\frac{1}{x} - \frac{x}{x^2+1}\right)dx$

$= \int \frac{1}{x}\,dx - \int \frac{x}{x^2+1}\,dx = \ln|x| - \int \frac{x}{x^2+1}\,dx$

let $u = x^2+1$
$du = 2x\,dx$

$= \ln|x| - \frac{1}{2}\int \frac{1}{u}\,du$

$= \ln|x| - \frac{1}{2}\ln|u| + C$

$= \boxed{\ln|x| - \frac{1}{2}\ln|x^2+1| + C}$

7.5 / 7.6 — 4 probs

① ② } Similar to probs from 7.1 to 7.4. You are just not told what method to use.

③ ④ } Table of Integrals

→ recognize the type

→ after a substitution you will use the table.

ex $\displaystyle\int \frac{x^2}{(3-\sqrt{2}\,x)^2}\,dx$

table $\displaystyle\int \frac{x^2}{(a+bx)^2}\,dx = \frac{1}{b^3}\left(a+bx - 2a\ln(a+bx) - \frac{a^2}{a+bx}\right)$

$$= -\frac{1}{2^{3/2}}\left(3-\sqrt{2}\,x - 6\ln(3-\sqrt{2}\,x) - \frac{9}{3-\sqrt{2}\,x}\right) + C$$

$\boxed{7.7}$ Numeric Approximation

ex $\displaystyle\int_0^1 \sqrt{1+x^4}\,dx$ use $n=4$ intervals

$\Delta x = \frac{1-0}{4} = 0.25$

$y=$	1	$\sqrt{1+.25^4}$	$\sqrt{1+.5^4}$	$\sqrt{1+.75^4}$	$\sqrt{2}$
$x=$	0	$.25$	$.5$	$.75$	1

$$L_4 = .25\left(1 + \sqrt{1+.25^4} + \sqrt{1+.5^4} + \sqrt{1+.75^4}\right)$$

$$R_4 = .25\left(\sqrt{1+.25^4} + \sqrt{1+.5^4} + \sqrt{1+.75^4} + \sqrt{2}\right)$$

$$T_4 = \frac{.25}{2}\left(1 + 2\sqrt{1+.25^4} + 2\sqrt{1+.5^4} + 2\sqrt{1+.75^4} + \sqrt{2}\right)$$

$$S_4 = \frac{.25}{3}\left(1 + 4\sqrt{1+.25^4} + 2\sqrt{1+.5^4} + 4\sqrt{1+.75^4} + \sqrt{2}\right)$$

$\boxed{7.8}$ Improper Integral (1 prob)

$$\int_a^b f(x)\,dx \to a \text{ and/or } b \text{ is } \pm\infty$$
$$\to f(x) \text{ has vertical asym. in } [a,b] \text{ interval}$$

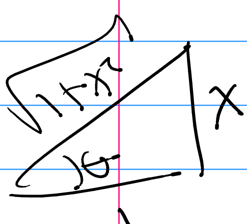
ex

$$\int_1^\infty \frac{1}{\sqrt{1+x^2}}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{\sqrt{1+x^2}}\,dx$$

let $x = \tan\theta$, $dx = \sec^2\theta\,d\theta$

$$\lim_{b\to\infty}\int_{x=1}^{x=b} \frac{\sec^{\cancel{2}}\theta\,d\theta}{\cancel{\sec\theta}}$$

$$= \lim_{b\to\infty}\int_{x=1}^{x=b} \sec\theta\,d\theta = \lim_{b\to\infty} \ln|\sec\theta + \tan\theta|\Big|_{x=1}^{x=b}$$



$$= \lim_{b\to\infty} \ln\left|\sqrt{1+x^2} + x\right|\Big|_{x=1}^{x=b}$$

$$= \lim_{b\to\infty}\left(\ln\left(\underline{\sqrt{1+b^2} + b}\right) - \ln\left(\sqrt{2}+1\right)\right) \to \infty$$

diverges

or arcsinh

or comparison

$$\frac{1}{\sqrt{1+x^2}} > \frac{1}{\sqrt{x^2+x^2}} = \frac{1}{\sqrt{2}\,x}$$

ex $\displaystyle\int_{-1}^{0} \frac{1}{x}\,dx = \lim_{a\to 0^-} \int_{-1}^{a} \frac{1}{x}\,dx$

$\displaystyle = \lim_{a\to 0^-} \ln|x| \Big|_{-1}^{a}$

$\displaystyle = \lim_{a\to 0^-} \ln|a| - \cancel{\ln|-1|}^{0} = \lim_{a\to 0^-} \ln|a| = -\infty$

$\boxed{\text{diverges}}$

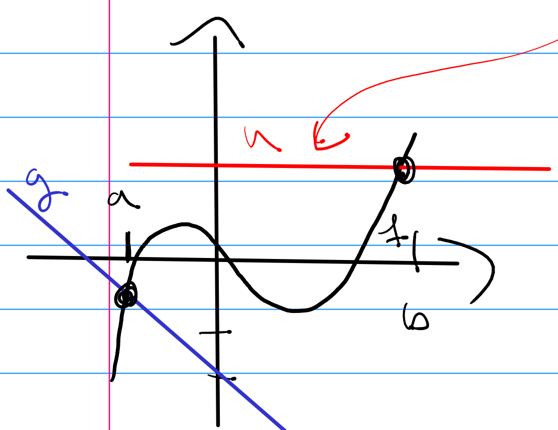
8.1 Arc length (2 probs)

(1) Set up

ex Arc length of $f(x) = x^3 - x$

between where it intersects $g(x) = -x - 2$

and $h(x) = 3$?



Solve: $-x-2 = x^3 - x$

$\to a = ?$

Solve: $3 = x^3 - x$

$\to b = ?$

$\rightarrow \quad L = \int_a^b \sqrt{1+(f')^2}\,dx \qquad f = x^3 - x$

$f' = 3x^2 - 1$

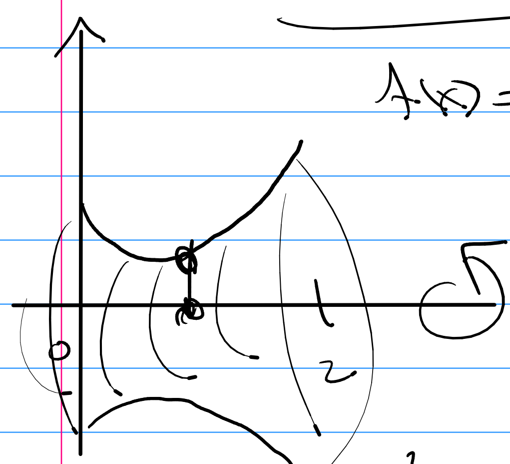
$$\boxed{L = \int_a^b \sqrt{1+(3x^2-1)^2}\,dx}$$

stop here.

② Do one.

8.2 Surface Area. (1 problem)

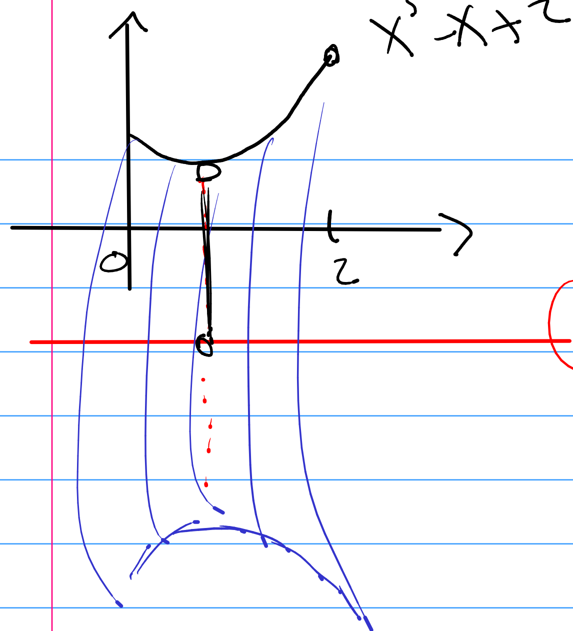
① Set it up only.



$f(x) = x^3 - x + 2$, between $x=0$ and $x=2$

$f'(x) = (3x^2-1)$

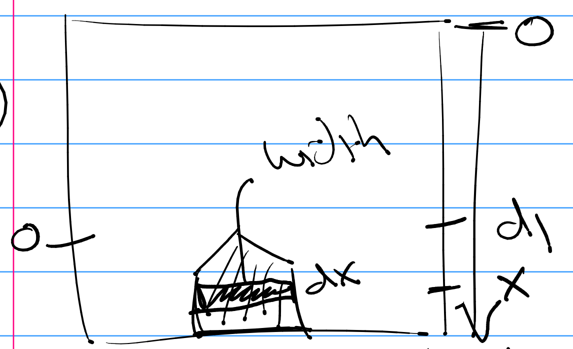
$$SA = \int_0^2 2\pi(x^3-x+2)\sqrt{1+(3x^2-1)^2}\,dx$$



$$SA=\int_0^2 2\pi\left(x^3-x+4\right)\sqrt{1+(3x^2-1)^2}\,dx$$

8.3 Hydrostatic Pressure / Force (1 prob)

ex



Find force on  shape

$$F=\int_{d_1}^{d_2}(\text{pressure})(\text{Area})$$

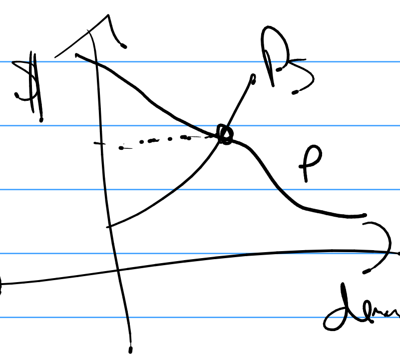
$$F=\int_{d_1}^{d_2}(\rho g x)(\text{width}\cdot dx)$$

8.4 Econ (1 prob)

ex given $p(x)$, $p_s(x)$

→ Find $P = p(X) = p_s(x)$

Requilibrium

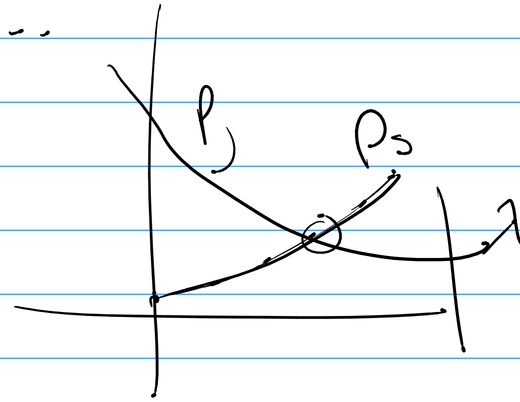


→ Find C.S. and P.S.

ex) Word Problem... blah blah blah...

$$p(x) = .1(x-10)^2$$

$$p_s(x) = .2x^2 + 2$$



(1) Equilibrium

$$.1(x-10)^2 = .2x^2 + 2$$

$$x^2 - 20x + 100 = 2x^2 + 20$$

$$x^2 + 20x - 80 = 0$$

$$x = \frac{-20 \pm \sqrt{400 + 320}}{2} = -10 \pm \frac{1}{2}\sqrt{720}$$

$$x = -10 + \frac{1}{2}\sqrt{720} \approx 3.4 \to \boxed{4 = x}$$

$$p(4) = .1(4-10)^2 = \boxed{\$3.6 = P}$$

(2) $CS = \int_0^4 \left(.1(x-10)^2 - 3.6\right) dx$

$PS = \int_0^4 \left(3.6 - (.2x^2 + 2)\right) dx$