Math 242

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x \qquad \Delta x = \frac{b-a}{n}$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} (b-a) \sum_{i=1}^{n} \frac{f(x_i^*)}{n}$$

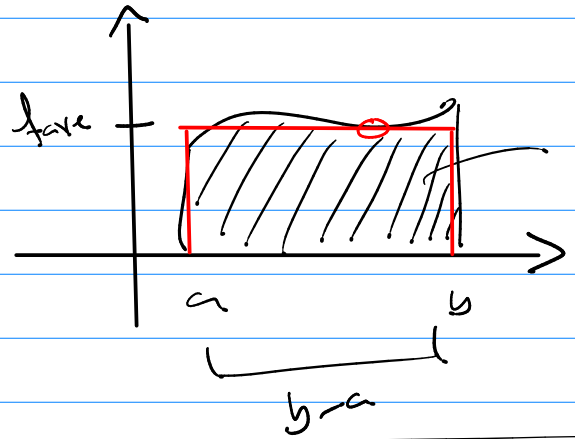
finite number

$$\int_a^b f(x)\,dx = (b-a) \lim_{n\to\infty} \left[\frac{\sum_{i=1}^{n} f(x_i^*)}{n} \right]$$

this is the arithmetic average

$$\int_a^b f(x)\,dx = (b-a)\, f_{ave}$$

Def: $f_{ave} = \frac{1}{(b-a)} \int_a^b f(x)\,dx$



$$A_{rea} = \int_a^b f(x)\,dx$$

$$A_{rea} = f_{ave} \cdot (b-a)$$

Mean Value th^m for integrals f is cont on $[a,b]$

then there exists a $x=c$ such that

$$f(c) = f_{ave} = \frac{1}{(b-a)} \int_a^b f(x)\,dx$$

or $(b-a)\, f(c) = \int_a^b f(x)\,dx$

App

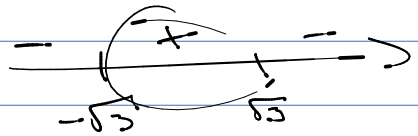
Favg ?

Ex $f(x) = \frac{x}{\sqrt{3-x^2}}$ over $[1,3]$

$$f_{avg} = \frac{1}{(b-a)} \int_a^b f(x)\,dx$$

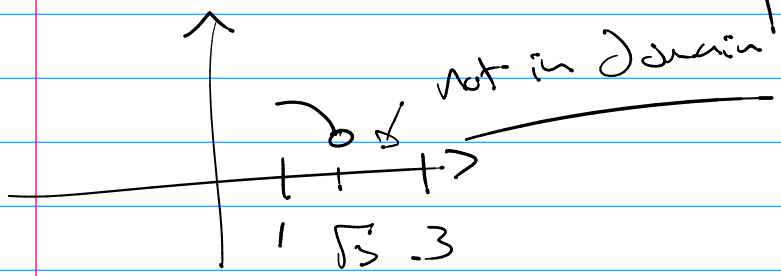
1st Domain? $3 - x^2 > 0 \rightarrow (-\sqrt{3}, \sqrt{3})$

$(\sqrt{3}+x)(\sqrt{3}-x) > 0$



2nd $f_{avg} = \frac{1}{b-a} \int_a^b f(x)\,dx$ $[a,b] = [1,3]$

not in Domain



Ex $f(x) = \frac{x}{\sqrt{3+x^2}}$ over $[1,3]$

1st Domain $3 + x^2 > 0$ true

2nd $f_{avg} = \frac{1}{3-1} \int_1^3 \frac{x}{\sqrt{3+x^2}}\,dx$

$$f_{ave} = \frac{1}{2}\int_1^3 \frac{x}{\sqrt{3+x^2}}\,dx = \frac{1}{4}\int_4^{12} u^{-1/2}\,du$$

let $u = 3+x^2$ $\quad x=1 \to u=4$

$du = 2x\,dx$ $\quad x=3 \to u=12$

$$= \frac{1}{2}u^{1/2}\Big|_{u=4}^{u=12} = \frac{1}{2}\left(\sqrt{12}-\sqrt{4}\right)$$

$$= \frac{1}{2}\left(2\sqrt{3}-2\right) = \sqrt{3}-1$$

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by MV thm of integrals

$x=c$ $\quad \frac{c}{\sqrt{3+c^2}} = \sqrt{3}-1 \to \frac{c^2}{3+c^2} = 4-2\sqrt{3}$

let $k = 4-2\sqrt{3}$

$\to c^2 = (4-2\sqrt{3})(3+c^2) \to c^2 = k(3+c^2)$

$\to c^2 - kc^2 = 3k \to c = \pm\sqrt{\frac{3k}{1-k}}$

$$\boxed{c = \sqrt{\frac{3(4-2\sqrt{3})}{2\sqrt{3}-3}}}$$

(ex) f_{ave}? $\quad f(t) = \cos^4 t \sin t \quad [0,\pi]$

$$f_{ave} = \frac{1}{\pi-0}\int_0^{\pi} (\cos t)^4 \sin t\,dt = \frac{-1}{\pi}\int_1^{-1} u^4\,du$$

let $u = \cos t$ $\quad t=0 \; u=1$

$du = -\sin t\,dt$ $\quad t=\pi \; u=-1$

$$f_{ave} = \frac{1}{\pi}\int_{-1}^{1} u^4\,du = \frac{1}{\pi}\left(\frac{1}{5}u^5\right)\Big|_{u=-1}^{u=1}$$

$$= \frac{1}{\pi}\left[\frac{1}{5} - \frac{-1}{5}\right] = \boxed{\frac{2}{5}\frac{1}{\pi}}$$

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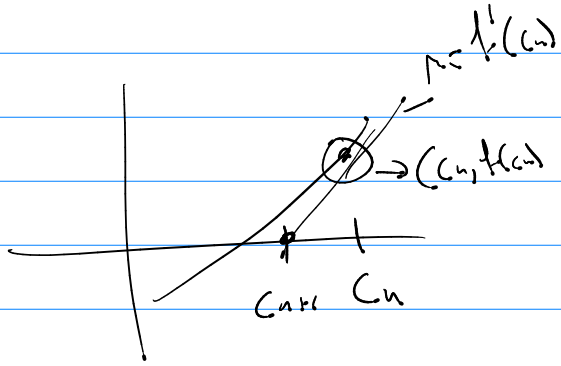
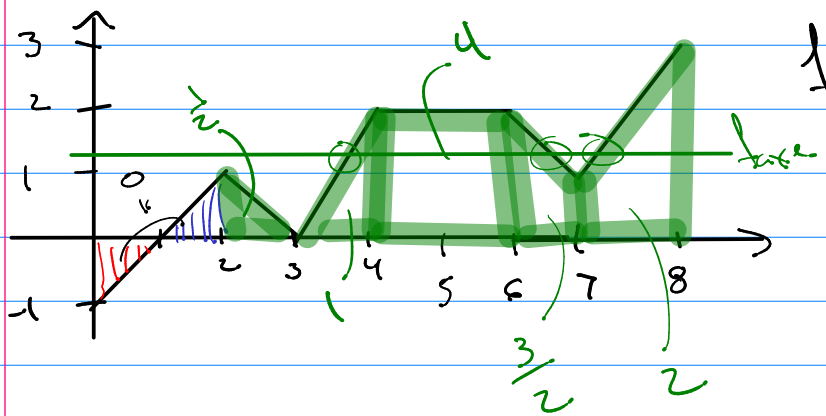
(12) $f(c) = \dfrac{2}{5\pi}$ $\qquad \cos^4(c)\sin(c) = \dfrac{2}{5\pi}$

$$\rightarrow \underbrace{\cos^4(c)\sin(c) - \frac{2}{5\pi} = 0}_{f(c) = 0}$$



① guess c_1

② $c_{n+1} = c_n - \dfrac{f(c_n)}{f'(c_n)}$



f_{ave} over $[0, 8]$

$$f_{ave} = \frac{1}{8}\int_0^8 f\,dx$$

↑ net signed area

$$f_{ave} = \frac{1}{8}\left(\frac{1}{2} + 1 + 4 + \frac{3}{2} + 2\right) = \boxed{\frac{9}{8}}$$

(R6) rod with linear density $\dfrac{12}{\sqrt{x+1}}$ $\dfrac{kg}{m}$, 8 m long.

density$_{ave}$ = ?

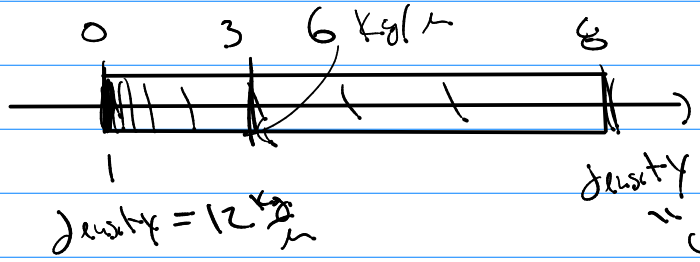
$$f_{ave} = \frac{1}{8-0}\int_0^8 \frac{12}{\sqrt{x+1}}\,dx$$

let $u = x+1$ $\quad x=0 \to u=1$ $\quad f_{ave} = \frac{1}{8}\int_1^9 \frac{12}{\sqrt{u}}\,du$

$du = dx$ $\quad x=8 \to u=9$

$$\to f_{ave} = \frac{3}{2}\int_1^9 u^{-1/2}\,du = 3\,u^{1/2}\Big|_{u=1}^{u=9}$$

$$= 3(\sqrt{9} - \sqrt{1}) = \boxed{6} \text{ kg/m}$$



$$\text{density}(x) = \frac{12}{\sqrt{x+1}}$$

When (c?) is density $= 6$ kg/m

$$\frac{12}{\sqrt{x+1}} = 6 \to 2 = \sqrt{x+1} \quad \boxed{x = 3\text{ m}}$$

(ex) average velocity from t_1 to t_2

$$v_{ave} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v(t)\,dt$$

b/c $v(t) = s'(t)$ ← deriv. of position

$$v_{ave} = \frac{1}{t_2 - t_1}\, s(t)\Big|_{t_1}^{t_2}$$

$$v_{ave} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Exam 4 — 21 probs

Ch 4 Integration / Areas

4.1/4.2 $\text{Area} = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i = \int_a^b f(x)dx$

If $\int_a^b f(x)dx$ exists $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$

$\Delta x = \frac{b-a}{n}$

2 probs ① Area by limit

② Approximation (left, right, mid pt)

4.3–4.5 $\int f(x)dx = F(x) + C$

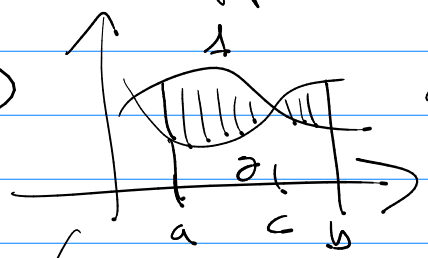
10 probs $\int_a^b f(x)dx = F(b) - F(a)$

Ch 5 Applications of integration

you will not solve all these →
- Setup only
- Setup + solve

5.1 area between curves.

2 probs



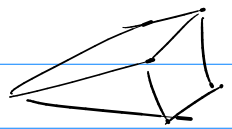
↑ Setup only

$\int_a^c (f-g)dx + \int_c^b (g-f)dx$

5.2/5.3 Volumes

3 probs

Slicing → ① generic object



→ volume of revolution

① Disks/washers

② Shells

5.4 Work

① Springs

② Water tanks

5.5 f_{ave}, $f(c) = f_{ave}$ $c = ?$

2 probs