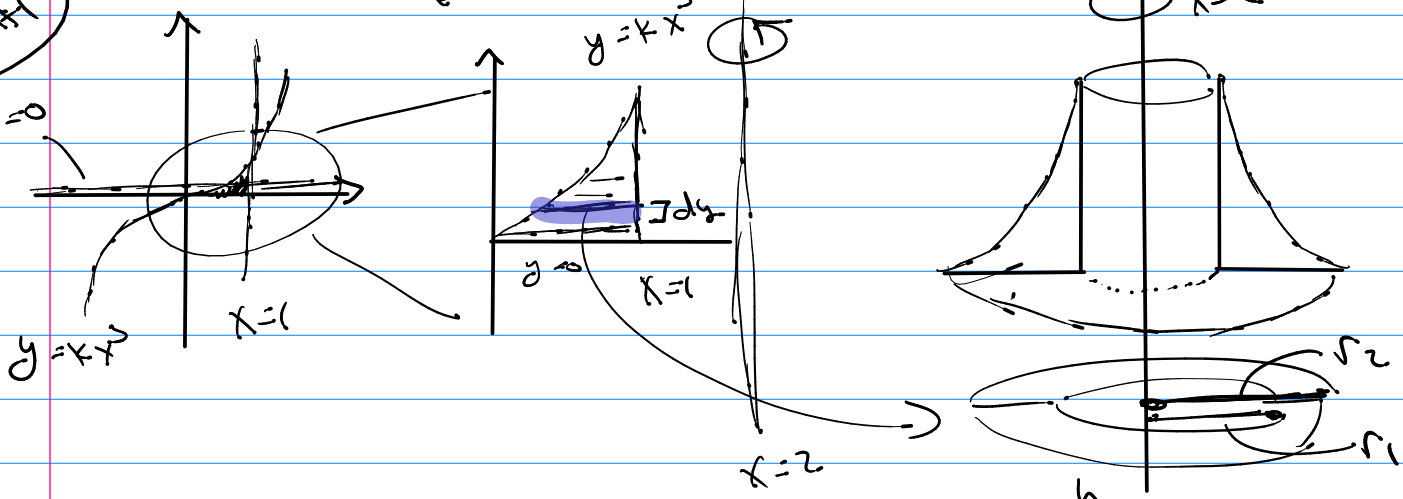
Math 242

Q's 5.2 #15 $y = kx^3$, $k > 0$, $y = 0$, $x = 1$ about $x = 2$

XY #1

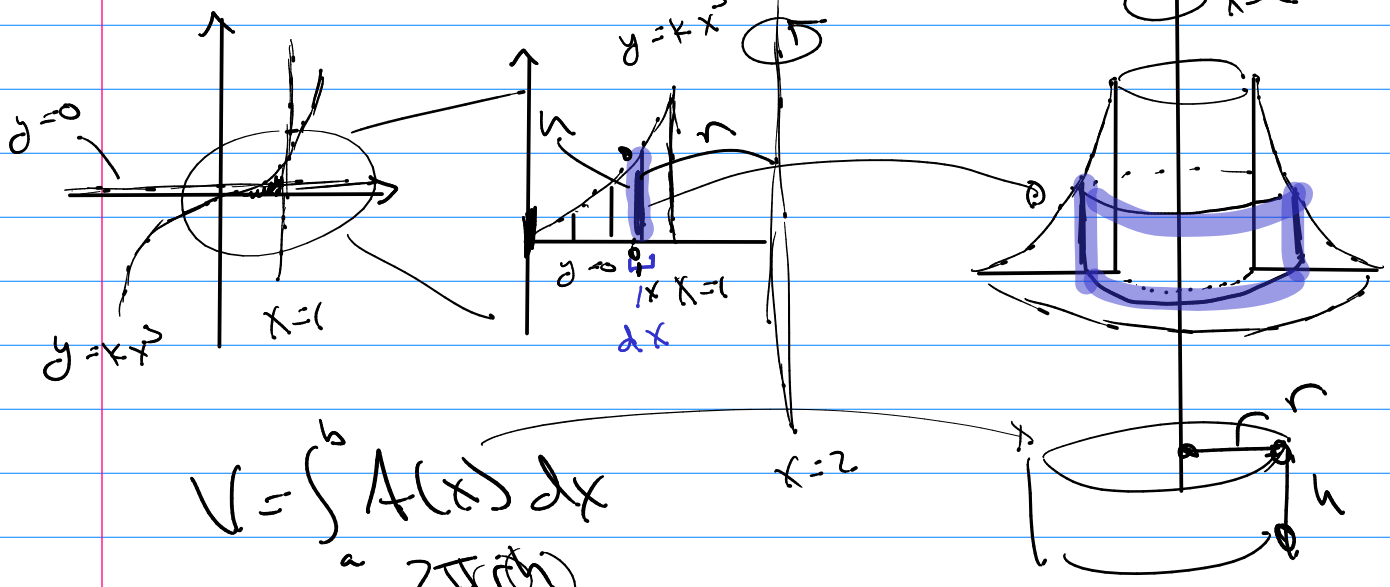


$$V = \int_a^b A(y)\,dy$$

$A(y) = \pi r_2^2 - \pi r_1^2$

CR

XY #2

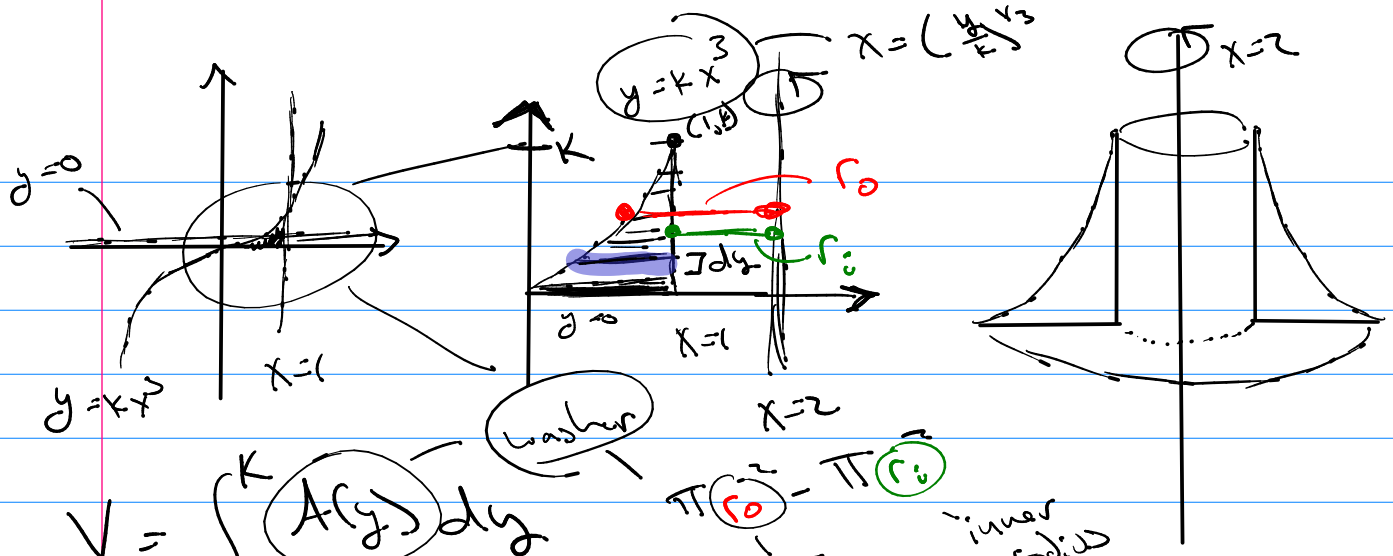


$$V = \int_a^b A(x)\,dx$$

$2\pi(\text{r})(\text{h})$

$$V = \int_0^1 2\pi(2-x)(kx^3)\,dx = 2k\pi \int_0^1 (2x^3 - x^4)\,dx$$

$$= 2k\pi\left(\frac{1}{2} - \frac{1}{5}\right) = \boxed{\frac{3}{5}k\pi}$$



Washer

$$V = \int_0^K A(y)\,dy$$

$\pi r_o^2 - \pi r_i^2$

r_o: outer radius

r_i: inner radius

$$V = \int_0^K \left(\pi\left(2 - \left(\frac{y}{k}\right)^{1/3}\right)^2 - \pi(2-1)^2\right)dy$$

$$V = \pi\int_0^K \left(\left(2 - \frac{1}{k^{1/3}}y^{1/3}\right)^2 - 1\right)dy = \text{Finish}$$

$$V = \pi\int_0^{27}\left(\left(2 - \frac{1}{3}y^{1/3}\right)^2 - 1\right)dy$$

$$V = \pi\int_0^{27}\left(3 - \frac{4}{3}y^{1/3} + \frac{1}{9}y^{2/3}\right)dy$$

$$V = \pi\left[3^4 - 3^4 + \frac{1}{5}3^4\right] = \frac{81}{5}\pi$$

$$\int f(x)\,dx = F(x) + C$$

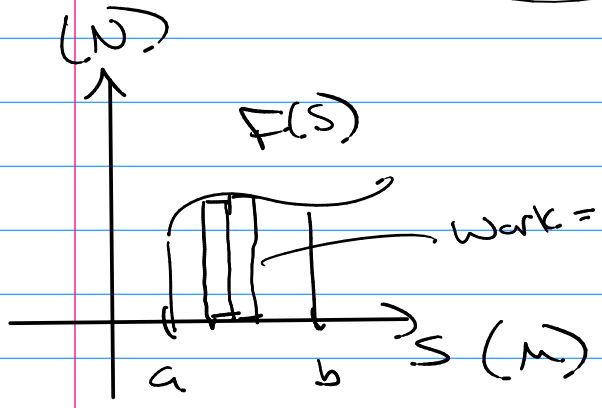
$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_{x=a}^{x=b} = F(b) - F(a)$$

Apps ① Area ② Volume ③ Work

5.4 Work = Force · displacement

Newton → Force = change in momentum

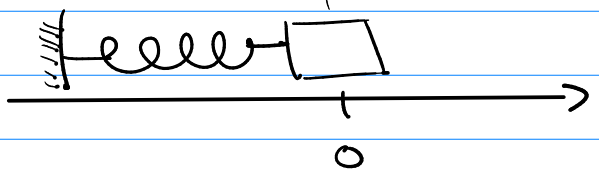
momentum = mass · velocity



work = area $W = \int_a^b F(s)\,ds$

Force

Example: Spring



$F_{spring} \propto s$

spring constant

spring at rest (natural displacement)

$\rightarrow F(s) = ks$

Ex: $s = 2\text{cm}$, $F = 3\text{N}$ (test)

$3\text{N} = k(.02\text{m}) \rightarrow 150\frac{\text{N}}{\text{m}} = k$

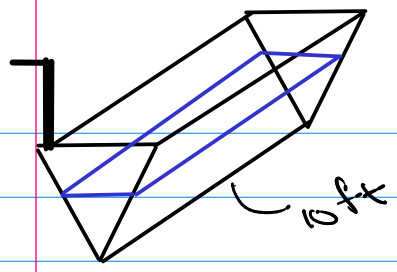
$F(s) = 150s$

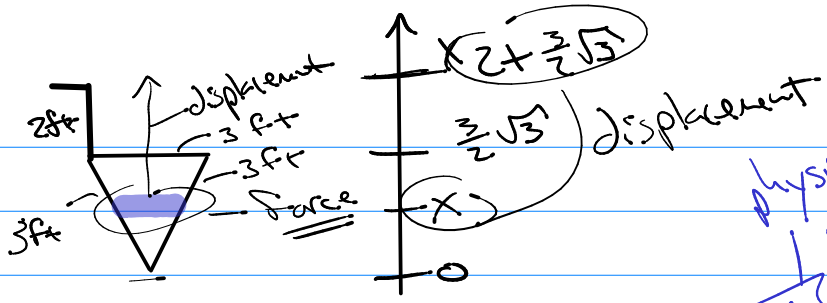
Ex:

Work to move it from rest to 20 cm.

$$W = \int_0^{.2} 150s\,ds = 75s^2\Big|_0^{.2} = 75(.2)^2 - 0$$

$$= \boxed{3\text{ Nm} = 3\text{ J}}$$





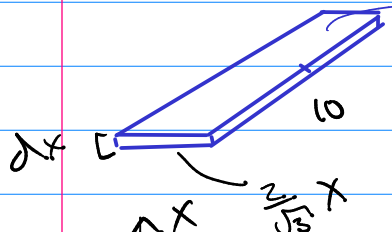
physical constant

ρ

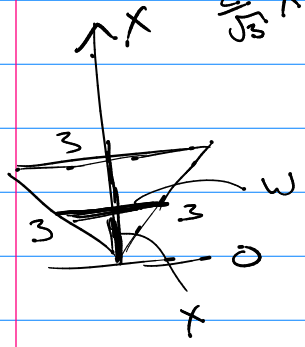
More this slice of water.

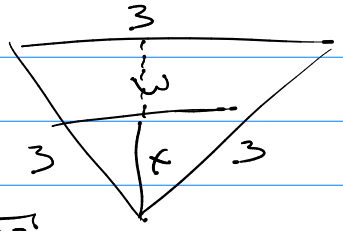
force = weight = Volume · Volume density

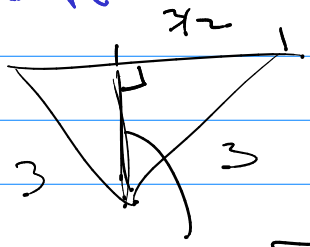
Volume density = $\frac{\text{weight}}{\text{vol.}}$



force = Volume · ρ $\frac{lb}{ft^3}$







$a^2 = 3^2 - \left(\frac{3}{2}\right)^2$

$a^2 = a\left(\frac{3}{4}\right)$

$a = \frac{3}{2}\sqrt{3}$

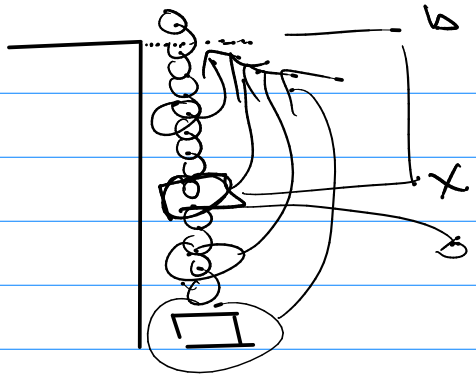
$\frac{x}{w} = \frac{\frac{3}{2}\sqrt{3}}{3}$

$w = \frac{2x}{\sqrt{3}}$

$$\text{Work} = \int_0^{\frac{3}{2}\sqrt{3}} \underbrace{\left((10)\left(\frac{2}{\sqrt{3}}x\right)dx\right)\rho}_{\text{weight} = \text{force}} \cdot \underbrace{\left(2+\frac{3}{2}\sqrt{3}-x\right)}_{\text{displacement}}$$

$$\text{Work} = \int_0^{\frac{3}{2}\sqrt{3}} \frac{20\rho}{\sqrt{3}} x\left(2+\frac{3}{2}\sqrt{3}-x\right)dx$$

Ex Chain

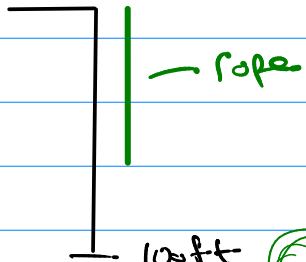


$b - x$

Work = weight · displacement

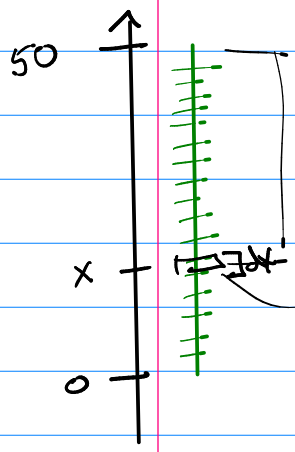
Ex 50 ft rope @ $\frac{1}{2}$ lb/ft hanging off building.

Work = lift it up.



Ex coiled rope @ bottom

Work = 2,500 ft lb



Displacement $= 50 - x$

weight $= \left(dx \frac{1}{2} \frac{lb}{ft}\right) = \frac{1}{2} dx$

$$\text{Work} = \int_0^{50} \left(\frac{1}{2} dx\right)(50 - x)$$

$$= \int_0^{50} \frac{1}{2}(50 - x)\,dx = \text{finish}$$