Math 242

Exam 3 $\to$ $\frac{1}{3}$ missed points fix

$f(x) = \dfrac{x}{x-1}$

$f'(x) = \dfrac{(x-1) - x}{(x-1)^2} = \boxed{-\dfrac{1}{(x-1)^2}} = -(x-1)^{-2}$

$f''(x) = 2(x-1)^{-3} = \dfrac{2}{(x-1)^3}$

$\boxed{f(x)}$ domain $x \neq 1$ vert. asym $x=1$

$\lim\limits_{x \to \pm\infty} \dfrac{x}{x-1} = 1$ horiz $y=1$

$\boxed{f'(x)}$ criticals $f'(x) = 0$ never $f'(x)$ dne $x=1$ asym

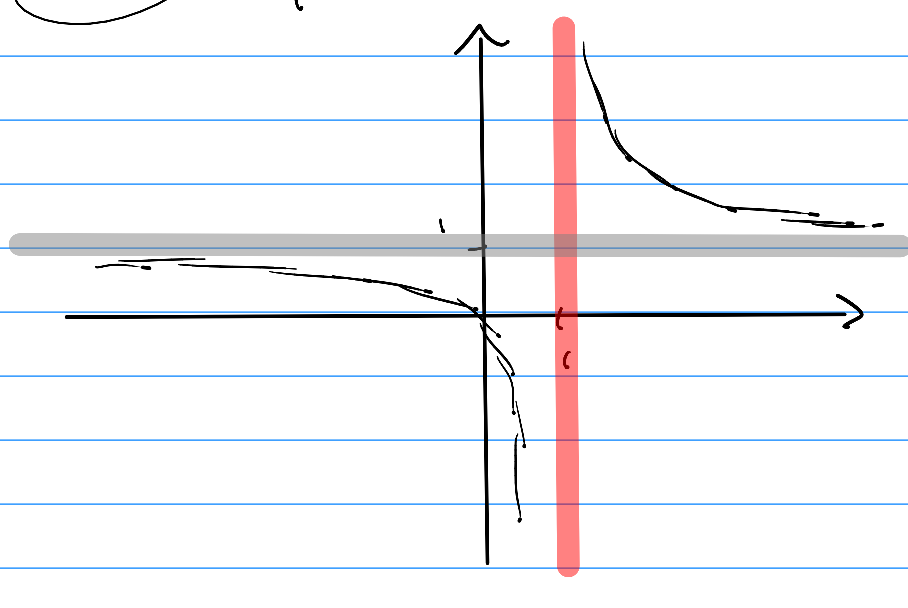
f': $-$ | 1 | $-$

extrema: none

$f''(x)$ $f''(x)=0$ never $f''(x)$ dne $x=1$ asym

f'': $-$ | 1 | $+$

Intercepts: $(0,0)$



$f(x) = (x-3)\sqrt{x} = x^{3/2} - 3x^{1/2}$

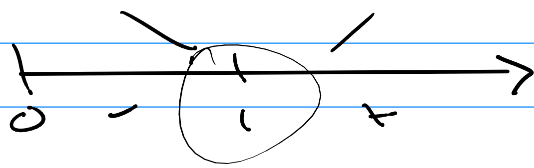
$f'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2} = \frac{3}{2\sqrt{x}}(x-1)$

$f''(x) = \frac{3}{4}x^{-1/2} + \frac{3}{4}x^{-3/2} = \frac{3}{4x^{3/2}}(x+1)$

$f(x)$ **intercepts:** $(0,0)$, $(3,0)$ asym. **none** domain $x \geq 0$

$f'(x)$ **criticals** $f'(x) = 0$: $x = 1$ $f'(x)$ dne: $x = 0$

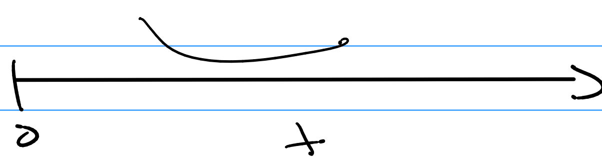
$f'(x)$

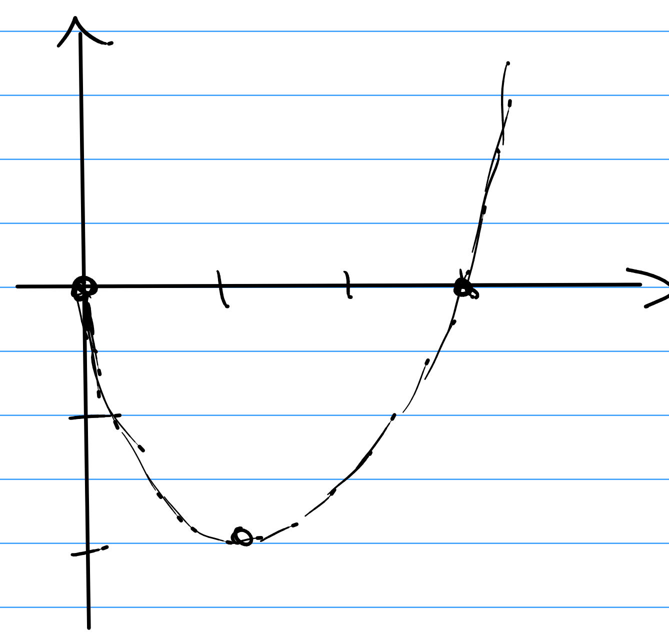


min @ $x=1$ & $f(1) = -2$

$f''(x)$ $f''(x) = 0$: never $f''(x)$ dne: $x = 0$

f''





$$f(x) = \frac{1}{x} - a \qquad f'(x) = -\frac{1}{x^2}$$

$$\boxed{r_{n+1} = r_n - \frac{f(r_n)}{f'(r_n)}} \rightarrow r_{n+1} = r_n - \frac{\frac{1}{r_n} - a}{-\frac{1}{r_n^2}}$$

$$r_{n+1} = r_n + r_n^2\left(\frac{1}{r_n} - a\right)$$

$$r_{n+1} = r_n + r_n - a r_n^2$$

$$\boxed{r_{n+1} = 2r_n - a r_n^2}$$

Till Today: Differential Calculus

$$D_x[f(x)] = f'(x)$$

(1) Know $\boxed{\text{all}}$ the rules

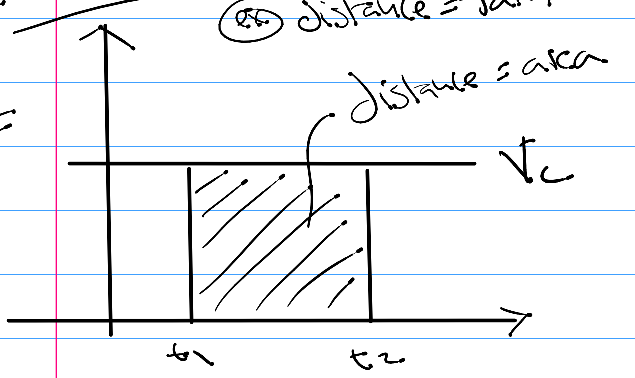
(2) limits

(3) Applications

Integral Calculus

(ex) distance = rate * time

Area:

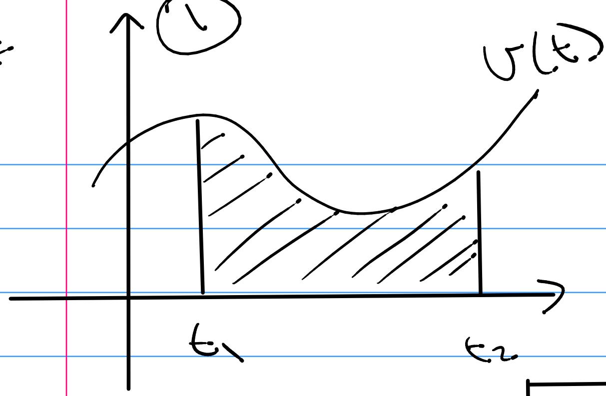


Antiderivatives

$$A_x[f(x)] = F(x) + C$$

when $D_x[F(x)] = f(x)$

Area

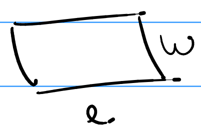


2

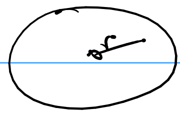
Anti-Derivative

$\boxed{\text{Fundamental thm of calculus}}$

area = distance

geometry: 

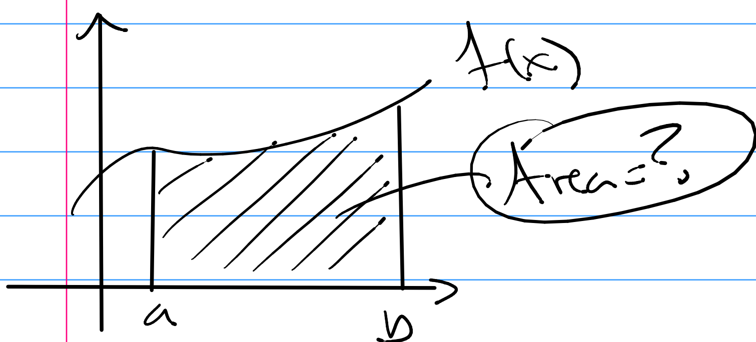
$A = l \cdot w$



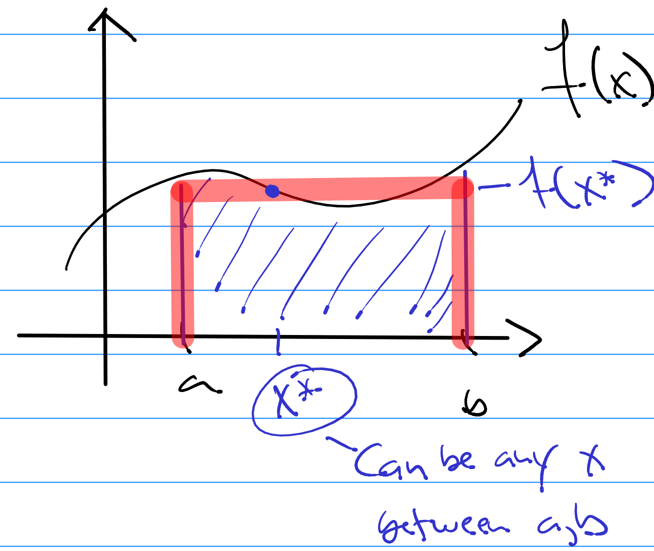
$A = \pi r^2$

---

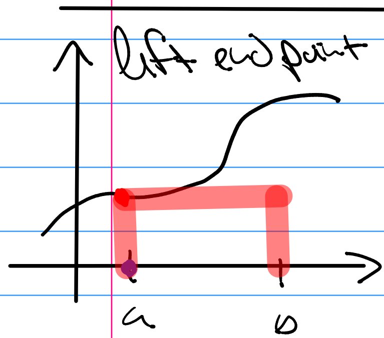
1 Areas under $f(x)$ over an interval.

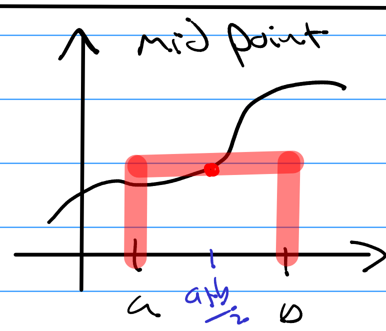


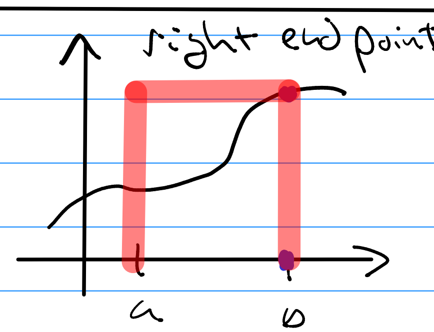
→ Can we approximate using rectangles?



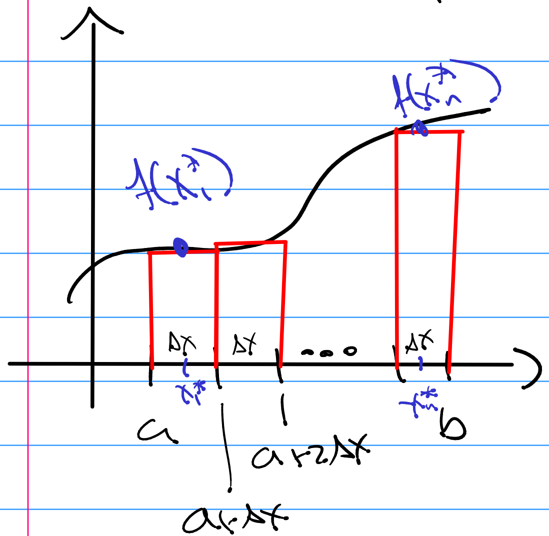
or take specific x^*







to make better approximates $\to$ use more rectangles



want n-rectangles

use divide up $[a,b]$ n-times

$$\Delta x = \frac{b-a}{n}$$

$$A \approx f(x_1^*)\Delta x + f(x_2^*)\Delta x + \dots + f(x_n^*)\Delta x$$

So $$A \approx \underbrace{\left(f(x_1^*) + f(x_2^*) + \dots + f(x_n^*)\right)\Delta x}_{R_n}$$

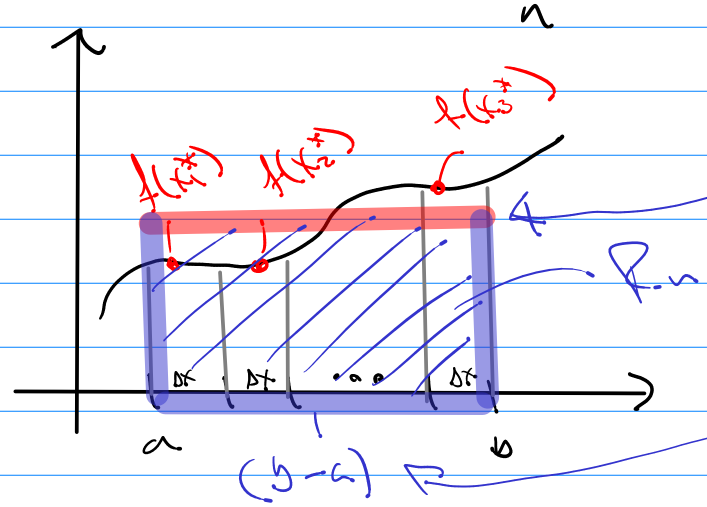
Now $$\lim_{n\to\infty} R_n = A$$

$$R_n = \left(f(x_1^*) + f(x_2^*) + \dots + f(x_n^*)\right)\Delta x$$

b/c $\Delta x = \frac{b-a}{n}$

$$R_n = \frac{\left(f(x_1^*) + f(x_2^*) + \dots + f(x_n^*)\right)}{n}(b-a)$$

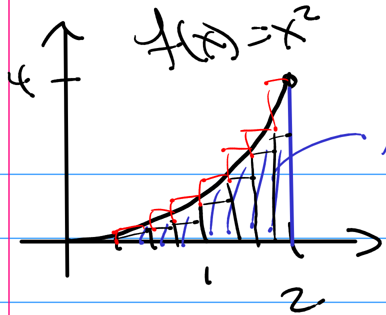
$\Delta x = \frac{b-a}{n}$



pick:

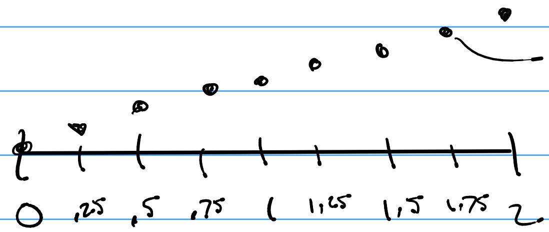
1. $x_i^* =$ left
2. $x_i^* =$ right
3. $x_i^* =$ midpoint

ex



Area under $f(x)=x^2$ over $[0,2]$

$n = 8 \qquad \Delta x = \frac{2-0}{8} = \frac{1}{4}$



$f(1.75) = (1.75)^2$

$$A = \lim_{n\to\infty} \left(f(x_1^*) + f(x_2^*) + \dots + f(x_n^*) \right) \Delta x$$

$\boxed{\text{New Notation}}$ Sequences, Sums

Sequence: $a_1, a_2, a_3, a_4, \dots = \{a_i\}$

& $a_n = f(n) \qquad = \{f(n)\}$

$\boxed{\text{So}}$ $f(x_1^*), f(x_2^*), \dots$

$\to \{f(x_i^*)\}$

$\boxed{\text{Sum}}$ = add up a sequence

$f(x_1^*) + f(x_2^*) + \dots + f(x_n^*)$

$= \sum_{i=1}^{n} f(x_i^*)$

ex $\boxed{\sum_{i=1}^{10} i^2} = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$

ex $\sum_{i=1}^{100} i = 1+2+3+\cdots+100 = \frac{100(101)}{2} = 50(101)$

$= \boxed{5050}$

$$\begin{array}{l} 1+2+3+4+\cdots+98+99+100 \\ 100+99+98+97+\cdots+3+2+1 \\ \hline 101+101+101+101+\cdots+101 = 100(101) \end{array}$$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Area under $f(x)$ over $[a,b]$

(1) $\Delta x = \frac{b-a}{n}$

(2) $\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$
