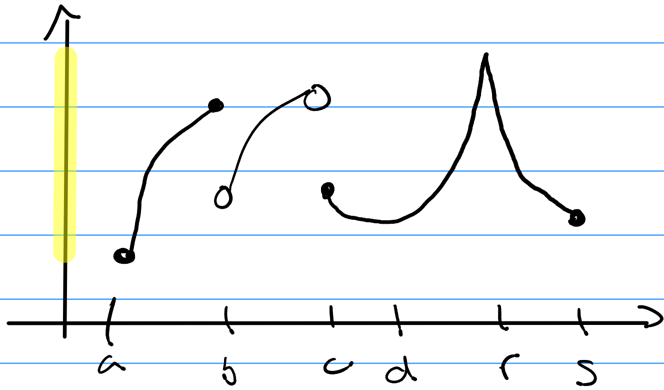
Math 242

Q5

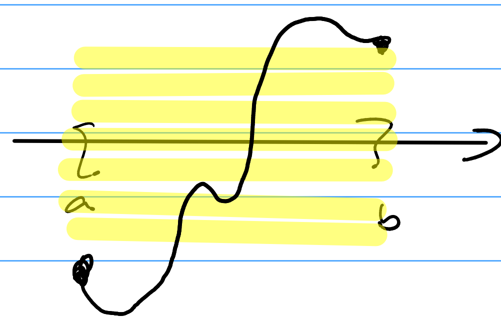


abs min = f(a) @ x=a
abs max = f(r) @ x=r
local min = f(d) @ x=d
local max = f(r) @ x=r

$\boxed{3.2}$ Mean Value thm

$\boxed{\text{Intermediate Value thm}}$

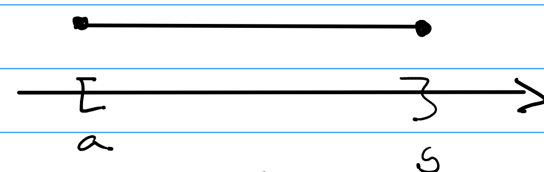
f is continuous on [a,b]



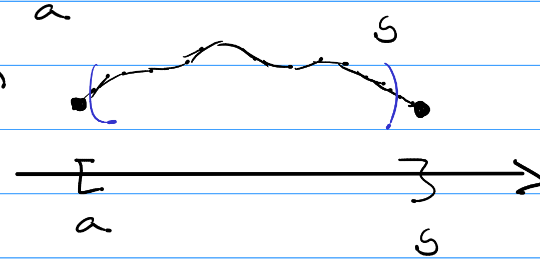
$\boxed{\text{Rolle's thm}}$

1. f(a) = f(b)
2. f is cont. on [a,b]
3. f' exists on (a,b)

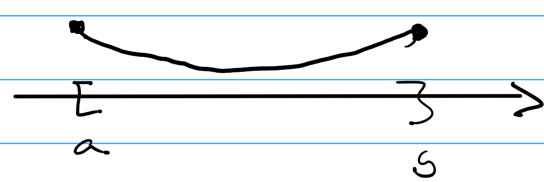
$\boxed{\text{Case 1}}$ f(x) = c



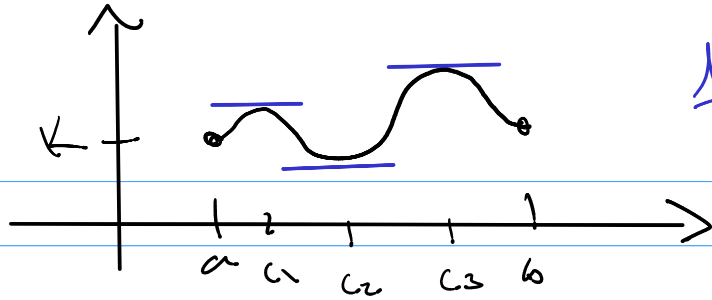
$\boxed{\text{Case 2}}$ f(x) > f(a) for some x in (a,b)



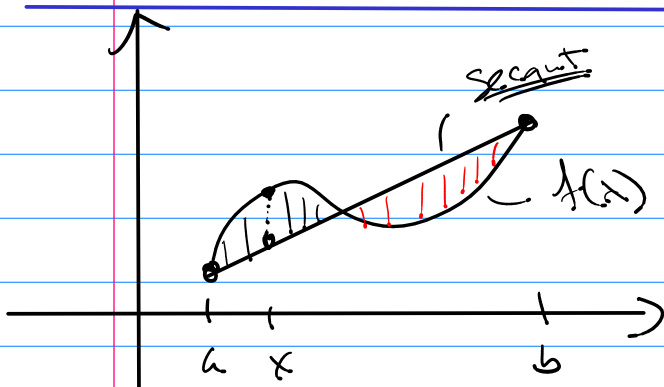
$\boxed{\text{Case 3}}$ f(x) < f(a) for some x in (a,b)



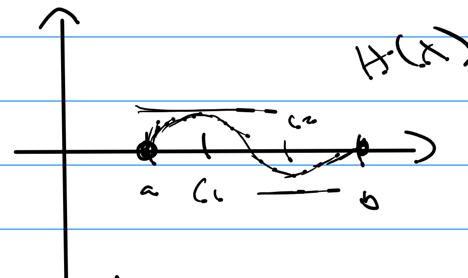
$\boxed{\text{then}}$ there is some c in (a,b) such that f'(c) = 0 ex



$f'(c_i) = 0$



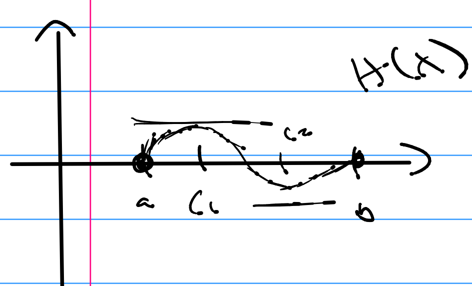
$H(x) = f(x) - \text{secant line}$



Secant line: $y - f(a) = \frac{f(b)-f(a)}{b-a}(x-a)$

$\to \boxed{y = f(a) + \frac{f(b)-f(a)}{b-a}(x-a)}$

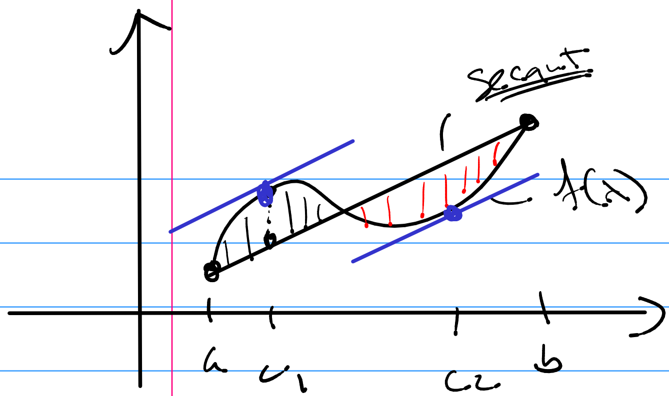
So $H(x) = f(x) - f(a) - \frac{f(b)-f(a)}{b-a}(x-a)$



Rolle's Applies

So $0 = H'(c) = f'(c) - \frac{f(b)-f(a)}{b-a}$

So $f'(c) = \frac{f(b)-f(a)}{b-a}$



$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

ave. velocity.

Mean Value Th^m

(1) f is cont on $[a,b]$
(2) f' exists on (a,b) (f is differentiable)

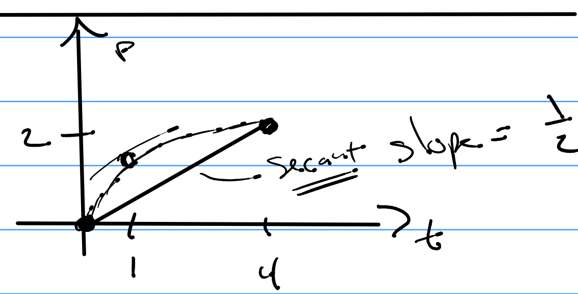
then there is a $c \in (a,b)$ such that

$$\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$$

or $\quad f(b)-f(a) = f'(c)(b-a)$

ex $\quad f(x) = x^{1/2} \quad$ on $\quad [0,4]$

$f'(x) = \frac{1}{2}x^{-1/2}$



$f'(c) = \frac{1}{2} \to \frac{1}{2\sqrt{c}} = \frac{1}{2} \to c=1$

@ $x=1 \quad f'(1) = \frac{f(4)-f(0)}{4-0}$

Mean Value th^m

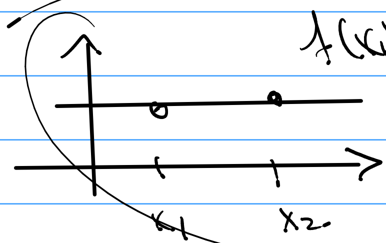
Another Application of Mean Value thm

if you take a Derivative .. can you undo it?

if $f'(x) = g'(x)$ $\rightarrow$ what do I know about f, g?

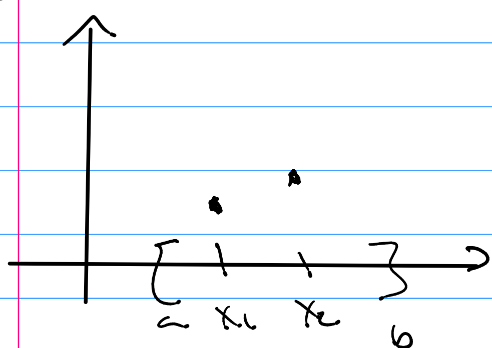
thm if $f'(x) = 0$ for all $x \in (a,b)$

$\rightarrow$ $f(x) = \text{constant}$



$f(x_1) = f(x_2)$ always

pf



any $x_1 < x_2$ in (a,b)

then: MVT

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(c)$$

$$\rightarrow f(x_2) - f(x_2) = \underbrace{f'(c)}_{0}(x_2 - x_1)$$

$$f(x_2) - f(x_1) = 0$$

$$f(x_2) = f(x_1) \quad \text{always}$$

So $f(x) = \text{constant}$

Corollary

if $f'(x) = g'(x)$ for all x in (a,b)

then: $\boxed{(f-g)(x)} = \text{constant}$ on all x in (a,b)

$f(x) - g(x) = c$ a constant

$f(x) = g(x) + c$

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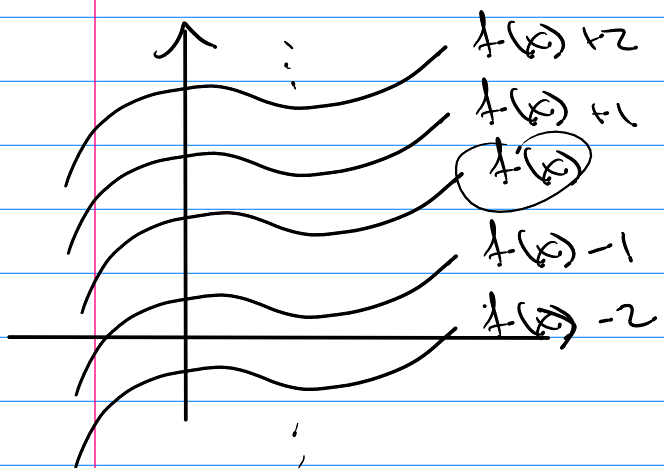
pf

$(f-g)(x) = f(x) - g(x)$ know $f'(x) = g'(x)$

$(f-g)'(x) = f'(x) - g'(x) = \underline{0}$

so $(f-g)(x) = \text{const}$



deriv. $\rightarrow$ $f'(x)$

Using Derivatives for graphing

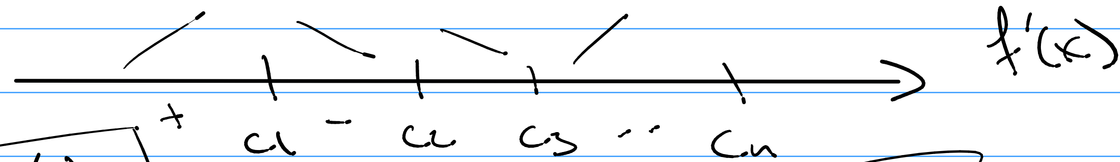
$f(x)$ points / position / asymptotes / Domain

$f'(x)$ slope / max / min

$f''(x)$ curvature / max / min

$f'(x)$

① $f'(x) = 0$, $f'(x)$ dne (corner / vert. asym)
critical numbers

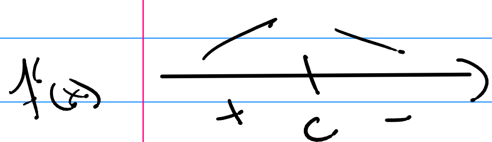
②


$\boxed{\text{inc / dec } f(x)}$

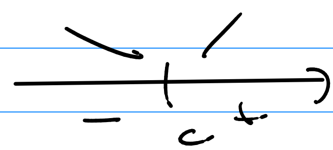
$f(x)$ inc. if $f'(x) \boxed{> 0}$

$f(x)$ dec. if $f'(x) < 0$

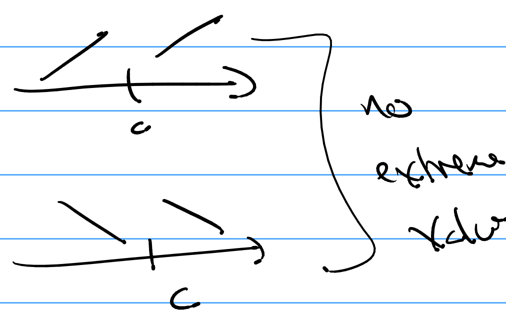
③ 1^{st} Deriv. test. ($f(c)$ exists)



local max
@ $x = c$

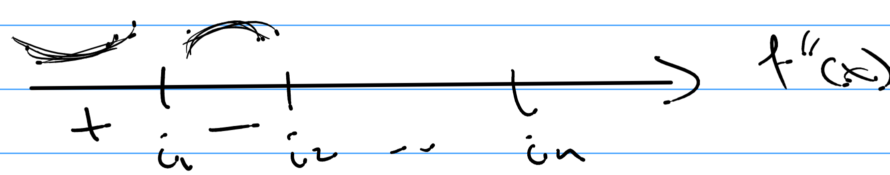


local min
@ $x = c$



$f''(x)$

① $f''(x) = 0$ or dne possible inflection point.

②


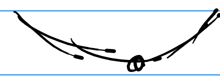
$f''(x) > 0$ concave up 

$f''(x) < 0$ concave down 

③ concavity switches → inflection point

4 2^{nd} Derivative test

c is a critical number, $f'(c) = 0$

$f''(c) > 0$,  $\rightarrow$ local min

$f''(c) < 0$,  $\rightarrow$ local max