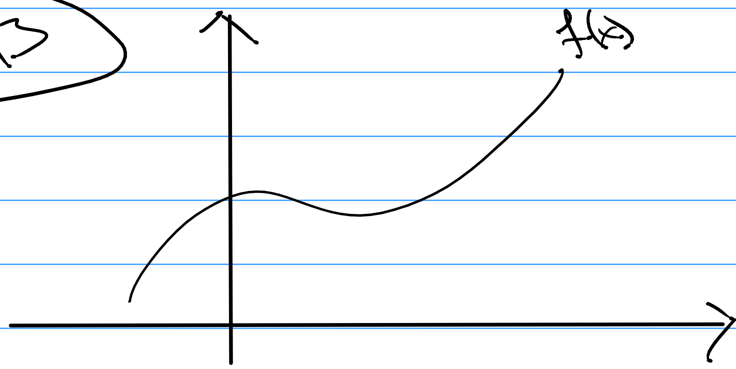
Math 242

Ch3



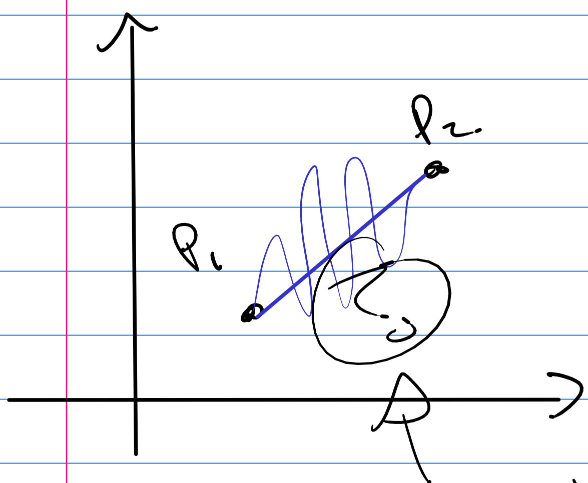
f(x)

x	y = f(x)

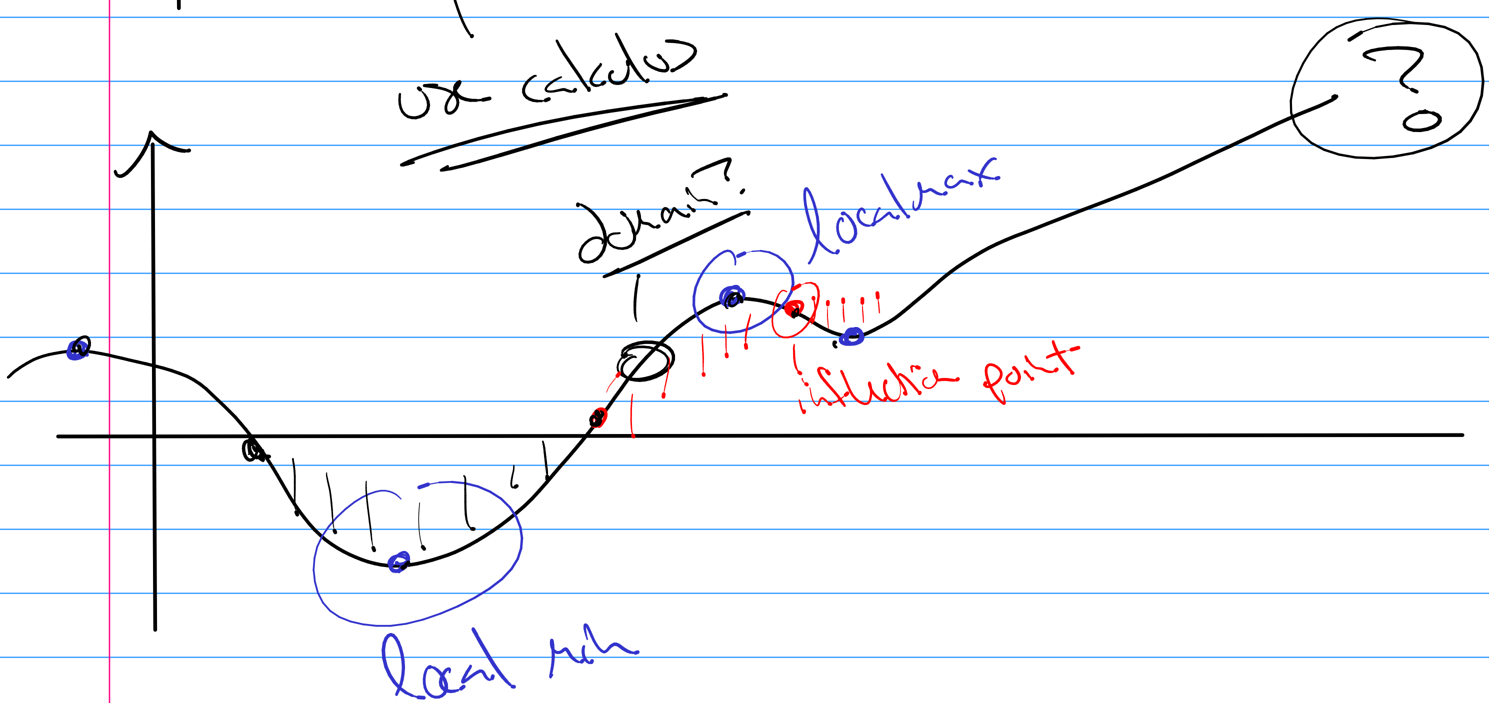
points

table

x	y
x_1	y_1
x_2	y_2

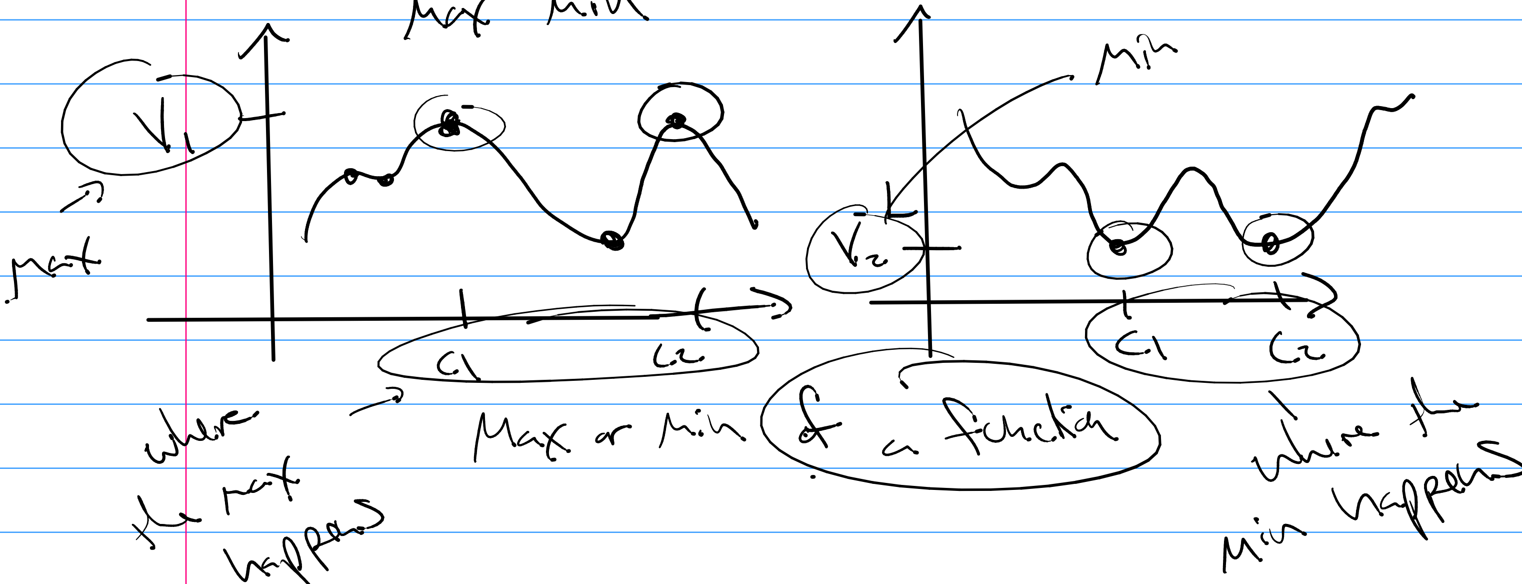


use calculus



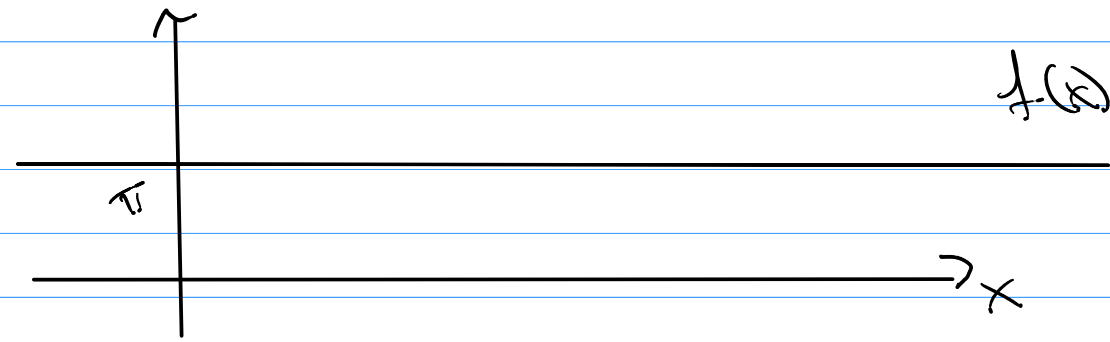
3.1 Extreme Values (Global, Local)

Max Min



ex

$f(x) = \pi$



Max $= \pi$ occurs @ all real values of x

Min $= \pi$ " " " " " " "

of Domain $= D$

Def (1) $f(c)$ is an absolute (global) max of f

max where

if $f(c) \geq f(x)$ for all x.

(2) $f(c)$ is an absolute min of f

min where if $f(c) \leq f(x)$ for all x.

Local

① $f(c)$ is a local max of f if $f(c) \geq f(x)$ near c.

② $f(c)$ is a local min of f if $f(c) \leq f(x)$ near c.

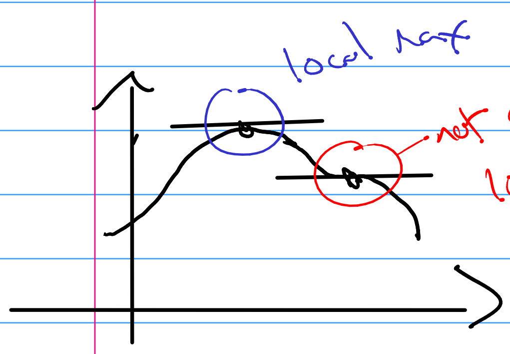
Where? → 1st question is do they exist?

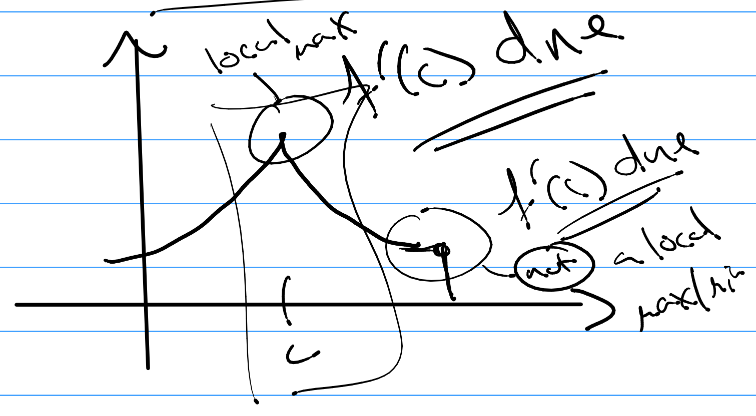
Thm if f is cont. on $[a,b]$ then $f(x)$ has an abs. max and an abs. min on $[a,b]$

Now onto where?

Thm (Fermat's thm)

if $f(x)$ has a local min or max @ $x=c$ and $f'(c)$ exists, then $\boxed{f'(c)=0}$





Where to look? critical numbers

all x such that $f'(x) = 0$
or $f'(x)$ dne (corners)

Check?

① Abs. Extrema on $[a, b]$

a) find $f'(x)$

b) find all c_i where $f'(c_i) = 0$
or $f'(c_i)$ dne (corner)

critical numbers

c) $f(a) = ?$
$f(b) = ?$ } end points

$f(c_i) = ?$ } critical points

max? = Max
min? = Min

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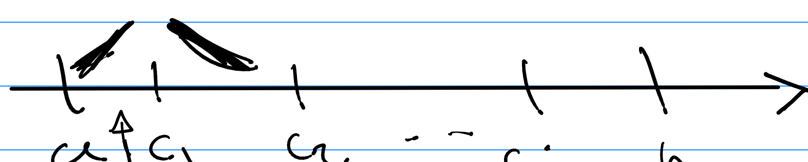
② Local extrema

1^{st} Derivative test

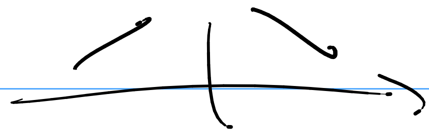
$f'(x)$



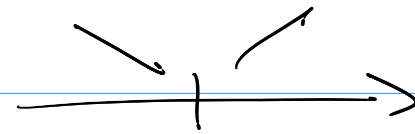
[f(any number in interval)] <0? >0?

local max:

f'

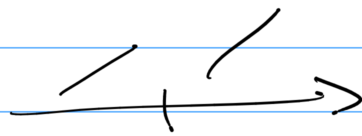


c.i.



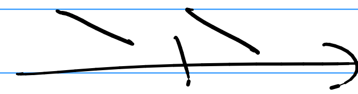
c.i.

local min



c.i.

nothing



c.i.

nothing

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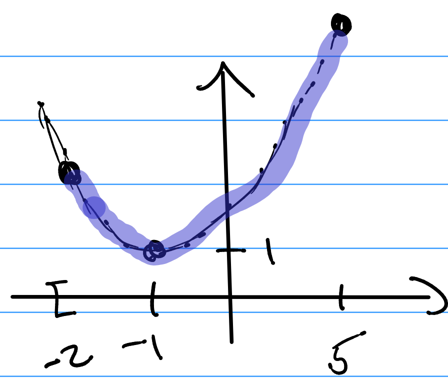
① $f(x) = 1 + (x+1)^2 \qquad x \in [-2, 5]$

abs min = 1
@ $x = -1$

abs max = 37
@ $x = 5$



$f(x) = (x+1)^2 + 1$

local min = 1
near $x = -1$

$f(x) = 1 + (x+1)^2$

$f'(x) = 2(x+1)^1(1) = 2x + 2$

Critical numbers

$f'(x) = 0 \rightarrow x = -1$

$f'(x)$ dne $\rightarrow$ never

abs extrema

$f(x) = 1 + (x+1)^2$

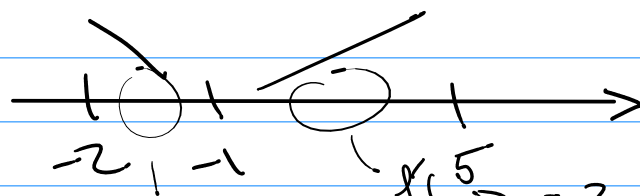
$f(-2) = 2$
$f(5) = 37$
$f(-1) = 1$

abs min $= 1$ @ $x = -1$

abs max $= 37$ @ $x = 5$

Local extrema

$f'(x) = 2x + 2$

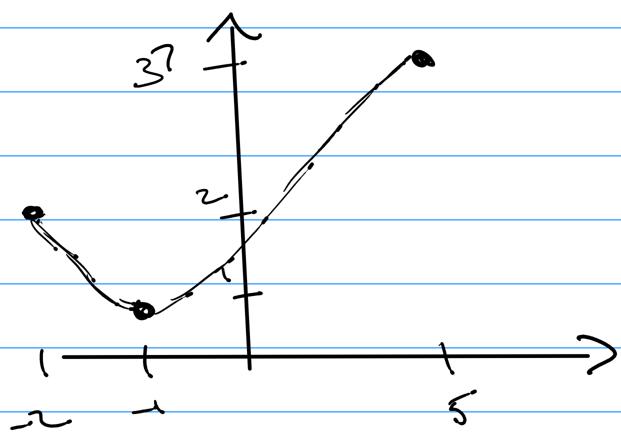


$f'(0) = 2$

$f'(-3/2) = -1$

so $\boxed{\text{local min @ } x = -1 \text{ \& } f(-1) = 1}$

Know



Ex $f(x) = \dfrac{x}{x^2 - x + 1}$ $[-3, 0]$

Natural Domain: $x^2 - x + 1 \neq 0$

$x^2 - x + 1 = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{1-4}}{2}$ -3

no real numbers

$$f(x) = \frac{x}{x^2-x+1}$$

$$f'(x) = \frac{(1)(x^2-x+1) - (x)(2x-1)}{(x^2-x+1)^2}$$

Simplify!

$f'(x) = 0$

$f'(x)$ dne never