Math 242

Implicit Derivatives

(1) Explicit vs Implicit Functions

explicit $h = h(s) = \dfrac{s^2 + \sin(s)}{\sqrt{s+1}}$

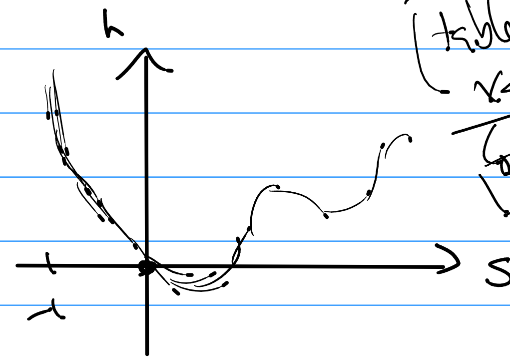


table of values
graph

independent variable: s
dependent variable: h

$$\frac{d}{ds}(h) = \boxed{D_s\left[\frac{s^2+\sin(s)}{(s+1)^{1/2}}\right]} = D_s\left[(s^2+\sin(s))(s+1)^{-1/2}\right]$$

quotient rule

product rule

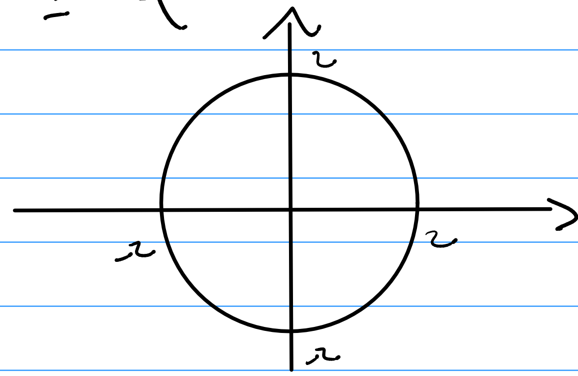
$$\frac{dh}{ds} = \frac{(2s+\cos(s))(s+1)^{1/2} - (s^2+\sin(s))\frac{1}{2}(s+1)^{-1/2}(1)}{(s+1)}$$

$$\frac{dh}{ds} = \frac{(s+1)^{-1/2}\left[(2s+\cos(s))(s+1)^{1} - \frac{1}{2}(s^2+\sin(s))\right]}{(s+1)^{1}}$$

$$\frac{dh}{ds} = \frac{2(2s+\cos(s))(s+1) - s^2 - \sin(s)}{2(s+1)^{3/2}}$$

Implicit Function:

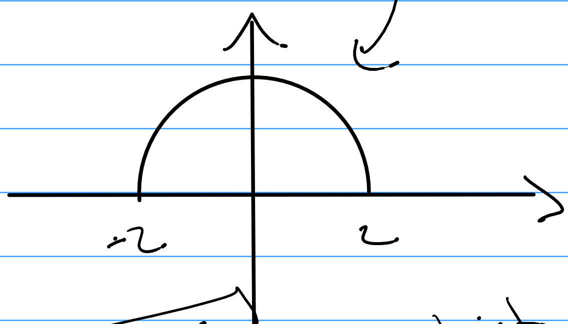
$$h^2 + s^2 = 4$$

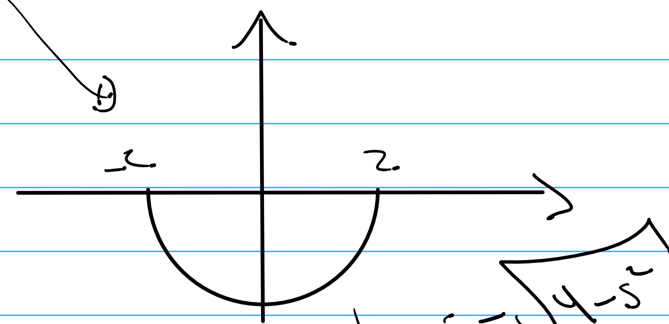


s: ind. var.

h: dep. var.

$h(s)$?





$h = +\sqrt{4-s^2}$ restrict domain and range $h = -\sqrt{4-s^2}$

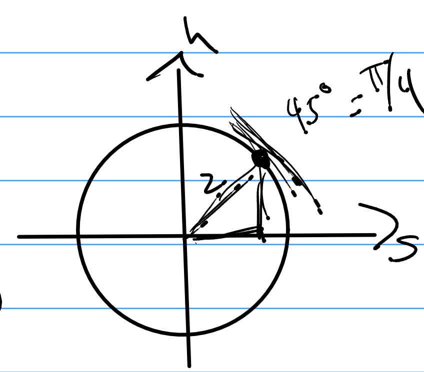
Use chain rule to find derivatives --

ex. $\boxed{h^2 + s^2 = 4}$ $\frac{dh}{ds} = ?$

$$D_s[h^2 + s^2] = D_s[4]$$

$$2h\boxed{\frac{dh}{ds}} + 2s = 0$$

$$\boxed{\frac{dh}{ds} = \frac{-s}{h}}$$



Point on $h^2 + s^2 = 4$

$s = 2\cos(\pi/4)$ $h = 2\sin(\pi/4)$

$$\frac{dh}{ds} = \frac{-2\cos(\pi/4)}{2\sin(\pi/4)} = -\cot(\pi/4) = -1$$

---

ex $x^2 + y^3 = 2xy$ eqn of tangent line

$D_x[x^2+y^3] = D_x[(2x)(y)]$ through $(.5118, .8447)$

slope? $\frac{dy}{dx} = ?$

so $2x + 3y^2\frac{dy}{dx} = (2)(y) + (2x)(1\cdot\frac{dy}{dx})$

$$3y^2\frac{dy}{dx} - 2x\frac{dy}{dx} = 2y - 2x$$

$$\frac{dy}{dx} = \frac{2y-2x}{3y^2-2x} \quad @ \ (.5118, .8447)$$

$$\frac{dy}{dx} = \frac{2(.8447) - 2(.5118)}{3(.8447)^2 - 2(.5118)} = m$$

$\boxed{\text{eqn of tangent line}}$ $\boxed{y - .8447 = m(x - .5118)}$

---

ex $x^2 + xy + y^2 = 3$ $y'' = ?$

$$y'' = \frac{d}{dx}\left[\frac{dy}{dx}\right]$$

$$D_x\left[x^2+(xy)+y^2\right]=D_x[3]$$

$$2x+\left[(1)(y)+(x)(1)\frac{dy}{dx}\right]+2y\frac{dy}{dx}=0$$

$$x\frac{dy}{dx}+2y\frac{dy}{dx}=-2x-y$$

$$\frac{dy}{dx}=-\frac{y+2x}{x+2y}$$

y''?

$$\frac{d}{dx}\left[\frac{dy}{dx}\right]= \text{''}$$

$$y''=-\frac{d}{dx}\left[\frac{y+2x}{x+2y}\right]=-\frac{\frac{d}{dx}[y+2x](x+2y)-(y+2x)\frac{d}{dx}(x+2y)}{(x+2y)^2}$$

$$y''=-\frac{\left(1\cdot\frac{dy}{dx}+2\right)(x+2y)-(y+2x)\left(1+2\cdot\frac{dy}{dx}\right)}{(x+2y)^2}$$

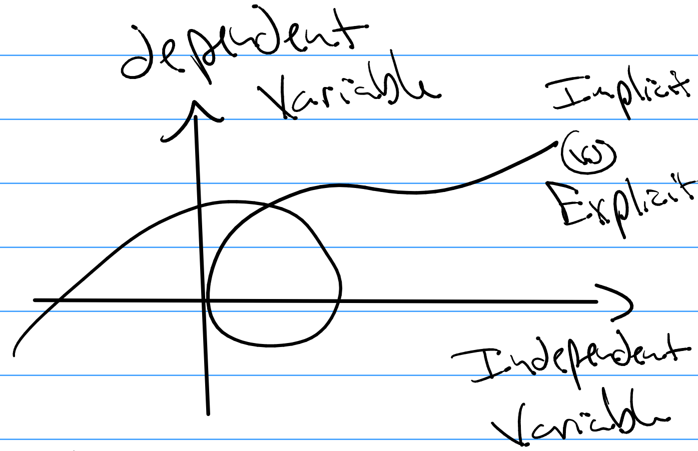
Note:
$$\frac{dy}{dx}=-\frac{y+2x}{x+2y}$$

$$y''=-\frac{\left(-\frac{y+2x}{x+2y}+2\right)(x+2y)-(y+2x)\left(1-2\left(\frac{y+2x}{x+2y}\right)\right)}{(x+2y)^2}$$

y'' = simplify

$\boxed{2.1 \to 2.6}$ how to take derivatives

Applications $\boxed{2.7}$



Idea: $y = f(x)$

Functions: relations / equations of values.

table

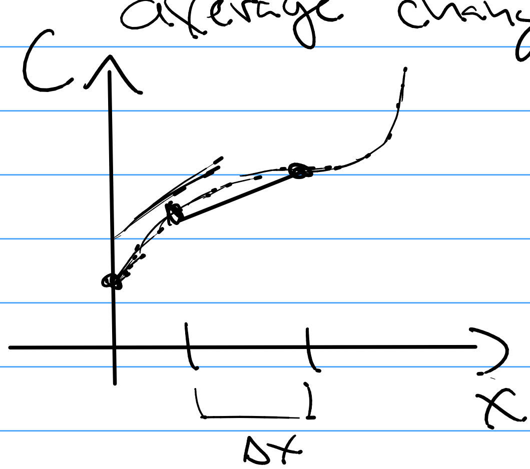
ind.	dep.
a	y $\to$ (a,b)

Derivatives: (change)

$\to \lim_{\Delta x \to 0} \boxed{\frac{\Delta y}{\Delta x}} = \boxed{\frac{dy}{dx}}$ instantaneous change

average change over $\Delta x = \frac{\Delta y}{\Delta x}$

ex



ave cost: $\frac{\Delta C}{\Delta x}$

$\frac{dc}{dx}$: marginal cost

(1) Physics: Position

$s = f(t)$

Velocity $s' = f'(t)$ vs $\frac{\Delta s}{\Delta t}$ aver. velocity

acceleration $s'' = f''(t)$

Looking forward: Force is what causes change in momentum.

(Mass)(velocity)

$\frac{d}{dt}\{m \cdot v\}$

$= m \cdot v' = m \cdot (\text{acceleration})$

(2) Current (Electricity)

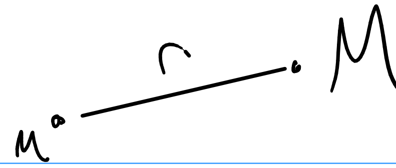
Function: charge $= Q(t)$

current $= \frac{dQ}{dt}$ ave. current $= \frac{\Delta Q}{\Delta t}$

(3) Cost / Revenue / Profit (econ)

ave Cost $= \frac{\Delta C}{\Delta x}$ marginal Cost $= \frac{dC}{dx}$

(20) $F = G\dfrac{mM}{r^2}$



$X \propto Y$ direct proportion

$X \propto \dfrac{1}{y}$ inverse proportion

(a) $\dfrac{dF}{dr} = D_r\left[\dfrac{GmM}{r^2}\right]$

$= GmM\, D_r\left[r^{-2}\right] = -\dfrac{2GmM}{r^3}$

$\dfrac{dF}{dr} = -\dfrac{2GmM}{r^3}$

inst. change in force with respect to distance.

$F = \dfrac{GmM}{r^2}$