Math 242

Functions

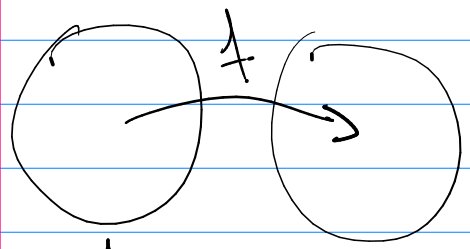
Relationships (rule to relate elements & several sets)

2 sets : Relation

Functions: $f : A \to B$ (two sets)

domain: A, codomain: B

function name: f



$a \to b$ $\quad (a, b)$

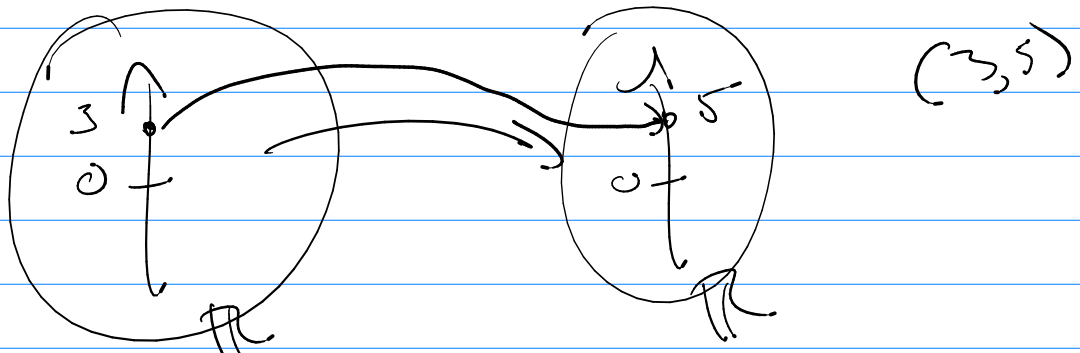
> f assigns each element of the domain exactly one element of the codomain.

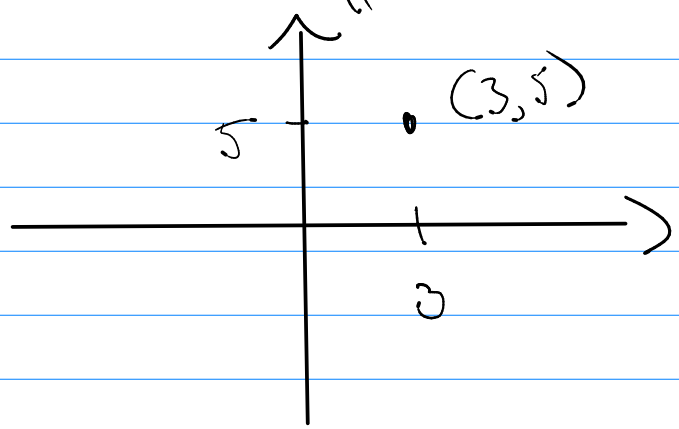
Calculus: $f : \mathbb{R} \to \mathbb{R}$

Notation: $\mathbb{R}$ (set of all real numbers)

$x \in \mathbb{R}$ $\quad x^3$ $\quad \mathbb{R}$

$f : \mathbb{R} \to \mathbb{R}$



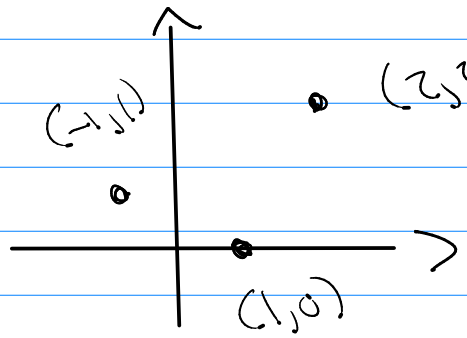


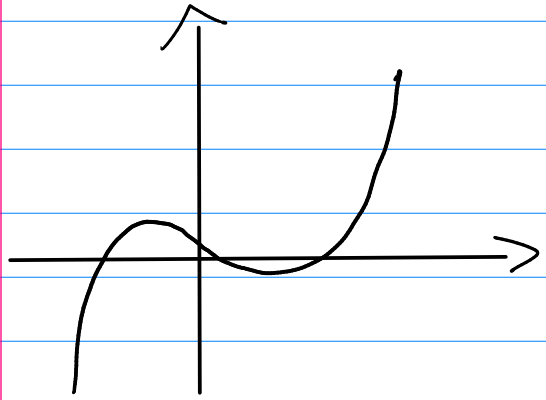
Representing $f : \mathbb{R} \to \mathbb{R}$

sets & ordered pairs

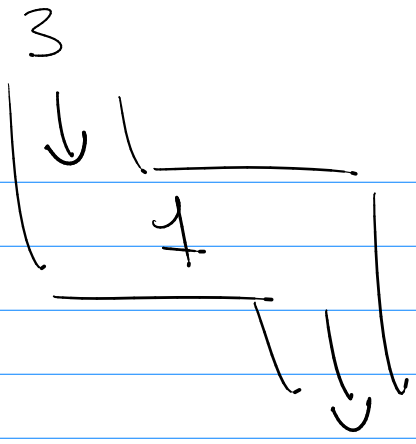
① table

② graph



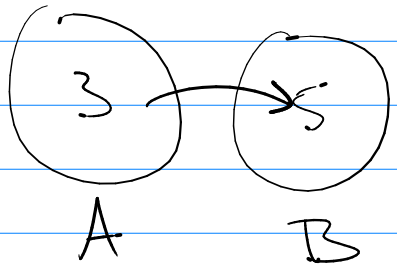


③ Machine diagram



④ Arrow diagram

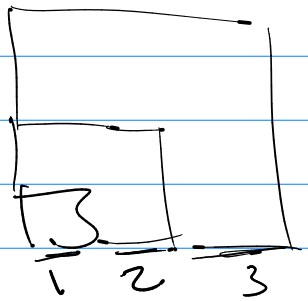
$f: A \to B$



Rules (why?)

① Verbal description

② Algebra



$3 \to 5$ b/c number, double minus 1

$$y = 2x - 1$$

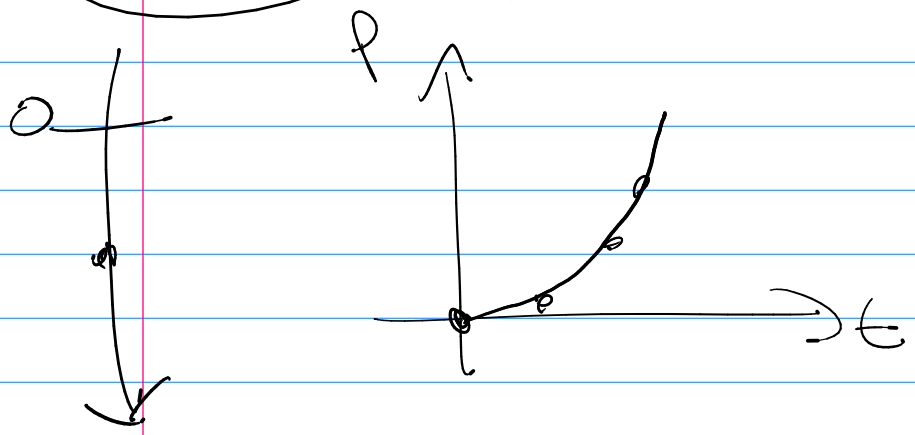
Notation: ex $f(x) = 3x^2 - 2x + 7$

$$f(\boxed{a+h}) = 3\boxed{a+h}^2 - 2\boxed{a+h} + 7$$

$$f(a+h) = 3(a+h)^2 - 2(a+h) + 7$$

$$= 3a^2 + 6ah + 3h^2 - 2a - 2h + 7$$

Part 3 Math (+) Models

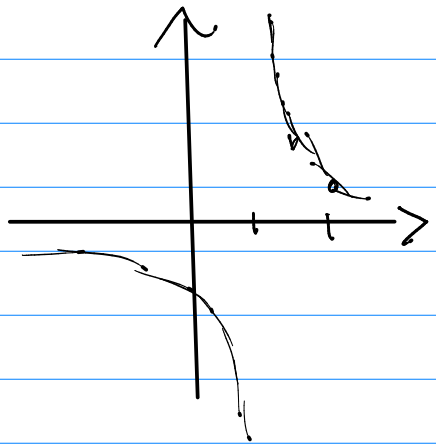


Graphs

$f: \mathbb{R} \to \mathbb{R}$

(1) Natural domain

ex $f(x) = \dfrac{1}{x-1}$



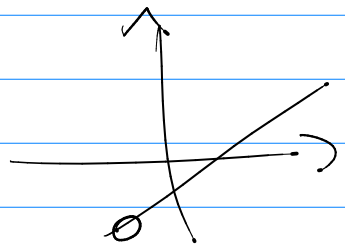
$x \neq 1$

domain = natural domain (all reals but $x \neq 1$)

ex $f(x) = \sqrt{x}$ domain: $x \geq 0$

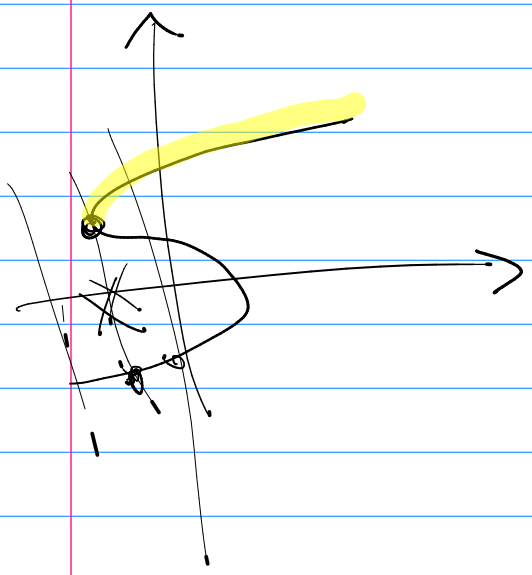
ex $f(x) = \dfrac{x^2-4}{x+2} = \dfrac{(x+2)(x-2)}{(x+2)} = x-2$

$x \neq -2$

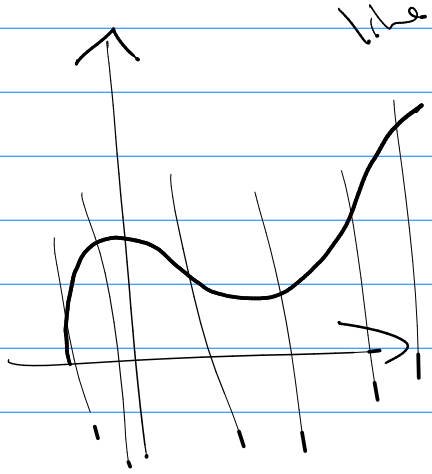


$f(x) = x - 2 \quad , \quad x \neq -2$

Is it a graph of function? vertical line test

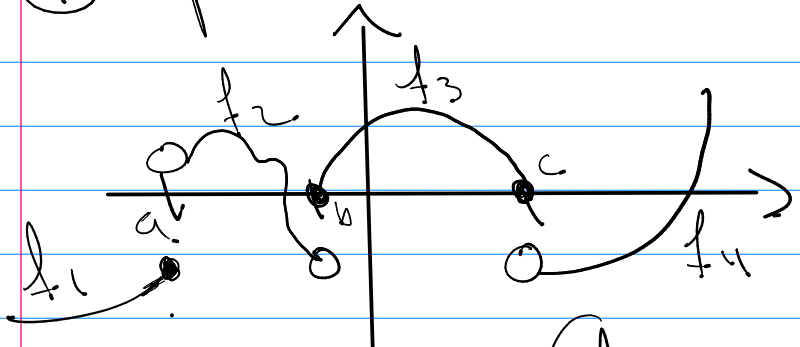


(x)



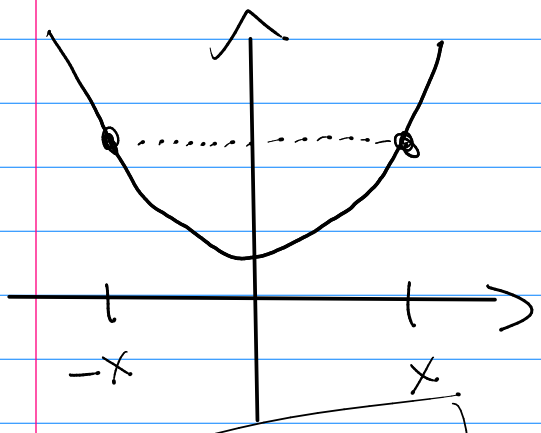
Types of functions

① piecewise defined



$$f(x) = \begin{cases} f_1(x) & x \leq a \\ f_2(x) & a < x < b \\ f_3(x) & b \leq x \leq c \\ f_4(x) & x > c \end{cases}$$

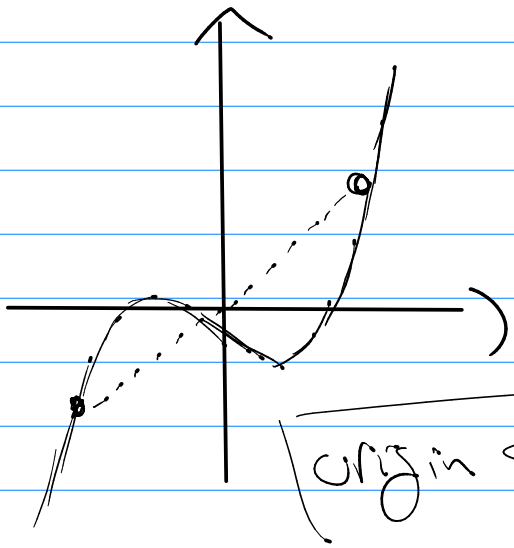
② by symmetry (y-axis sym. / origin sym)



y-axis sym

$f(-x) = f(x)$

even function

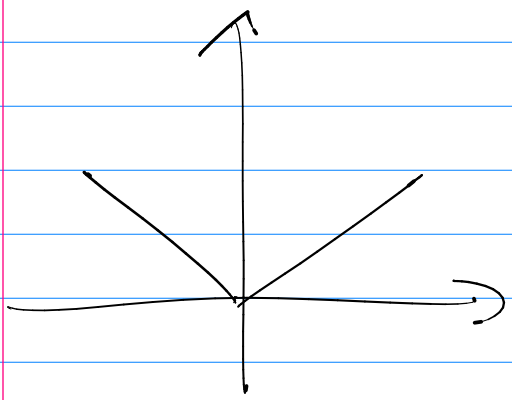


origin sym

$f(-x) = -f(x)$

odd function

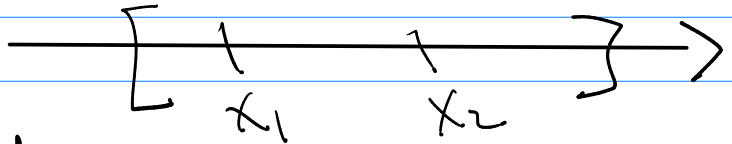
ex $|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$



(3) Inc / Dec

if x_1, x_2 on an interval

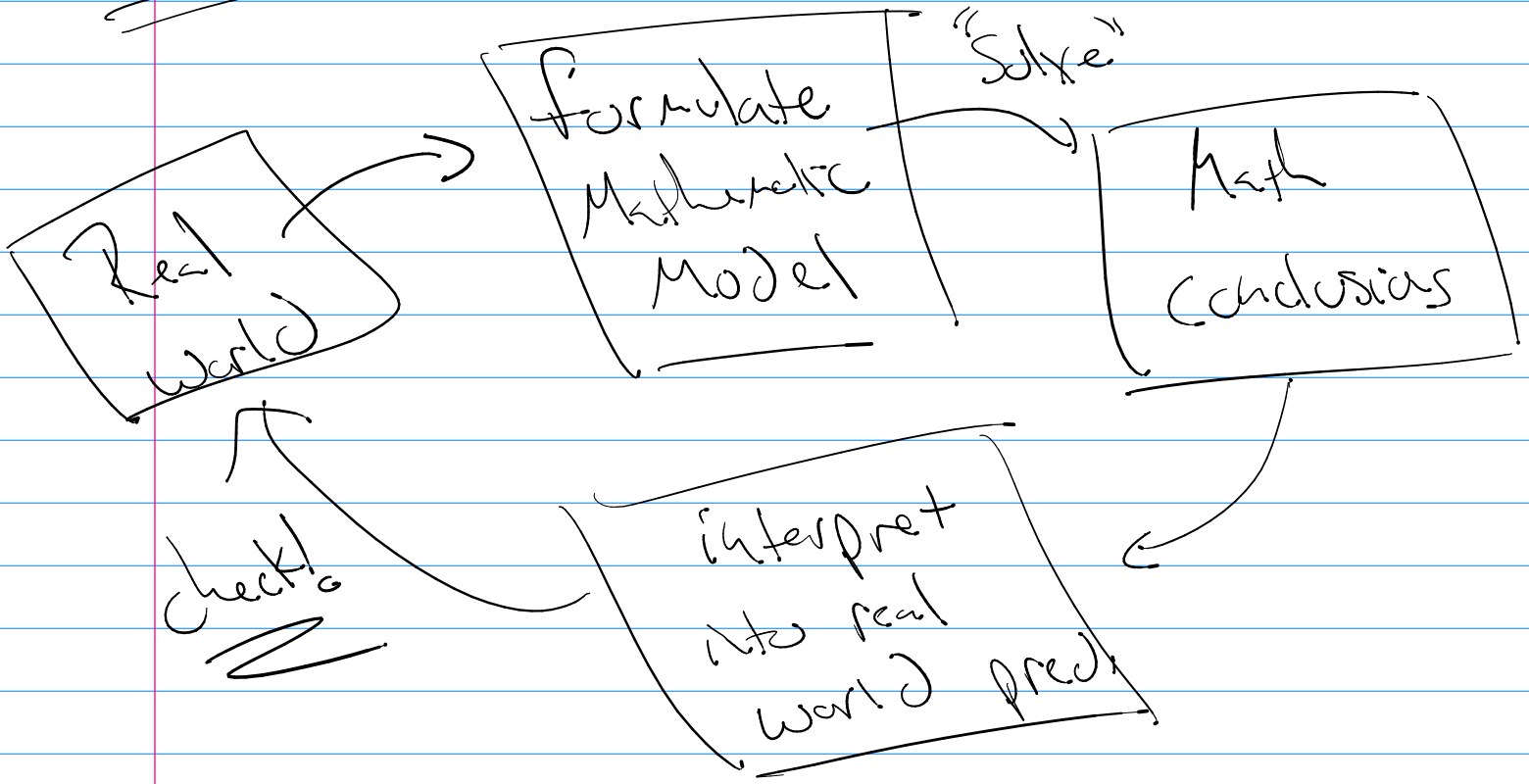
and $x_1 < x_2$



a) if $f(x_1) < f(x_2)$ call f Inc.

b) if $f(x_1) > f(x_2)$ call f Dec

Back to Modeling



Types of Functions

(1) Polynomial $p(x) = a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0$

degree $= 1 \rightarrow p(x) = ax + b$

degree $= 2 \rightarrow p(x) = ax^2 + bx + c$

$= 3 \rightarrow p(x) = ax^3 + bx^2 + cx + d$

$= 4 \rightarrow p(x) = ax^4 + bx^3 + cx^2 + dx + e$

(2) Power: $f(x) = x^p$ (p is some real)

i) $p = 0, 1, 2, 3, \ldots$ (polynomial)

ii) $p = \frac{1}{n}$ $n = 2, 3, 4, \ldots$

$p = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$f(x) = \sqrt{x} = x^{1/2}$

$= \sqrt[3]{x} = x^{1/3}$

$= \sqrt[p]{x} = x^{1/p}$

iii) $p=-1$ $\quad f(x)=\frac{1}{x}$

(3) Rationals: $\quad f(x)=\frac{\text{polynomial}}{\text{polynomial}}$

(4) Algebric $\quad (+,-,\times,\div,\text{Roots})$

$$f(x)=\frac{\sqrt{x+4}}{x^3-3x}+7\sqrt[7]{x}+\sqrt{11}$$

(5) Trigonometric

(6) Exponential $\quad f(x)=a^x$

(7) logarithmic $\quad f(x)=\log_a x$

$$y=\log_a(x)$$

$$a^y=x$$

$$\sqrt{x}=y$$

$$y\cdot y=x$$

$$y^2=x$$