Radial Limits of Bounded Nonparametric PMC Surfaces

Mozhgan Entekhabi & Kirk E. Lancaster
Department of Mathematics, Statistics & Physics
Wichita State University
Wichita, Kansas, 67260-0033

Dedicated to the memory of Alan Ross Elcrat

Abstract
Consider a solution \( f \in C^2(\Omega) \) of a prescribed mean curvature equation

\[
\text{div} \left( \frac{\nabla f}{\sqrt{1 + |\nabla f|^2}} \right) = 2H(x, f) \quad \text{in} \; \Omega,
\]

where \( \Omega \subset \mathbb{R}^2 \) is a domain whose boundary has a corner at \( C = (0, 0) \in \partial \Omega \). If \( \sup_{x \in \Omega} |f(x)| \) and \( \sup_{x \in \Omega} |H(x, f(x))| \) are both finite and \( \Omega \) has a reentrant corner at \( C \), then the (nontangential) radial limits of \( f \) at \( C \),

\[
Rf(\theta) \equiv \lim_{r \to 0} f(r \cos(\theta), r \sin(\theta)),
\]

are shown to exist, independent of the boundary behavior of \( f \) on \( \partial \Omega \), and to have a specific type of behavior. If \( \sup_{x \in \Omega} |f(x)| \) and \( \sup_{x \in \Omega} |H(x, f(x))| \) are both finite and the trace of \( f \) on one side has a limit at \( C \), then the (nontangential) radial limits of \( f \) at \( C \) exist, the tangential radial limit of \( f \) at \( C \) from one side exists and the radial limits have a specific type of behavior.