WICHITA STATE UNIVERSITY

Department of Mathematics and Statistics

The Lecture Series in the Mathematical Sciences Presents Our Guest:

Dr. Hari Mukerjee

Wichita State University

"Weak Convergence of Nonmeasurable Functions"

Abstract:

First, there was De Moivre-Laplace Central Limit Theorem (CLT) for binomial random variables (RV's). Then came CLT's for sample means of $X_1, X_2, \ldots, X_n$ of Identically and Independently Distributed (IID) RV's with distribution function (DF), F, i.e., a random sample from the DF, F ($F(x) = P(X_1 \leq x)$), with mean, μ, and a finite variance, σ^2, ($\bar{X}_n = \sum_{i=1}^n X_i/n$)

$$P\left(\sqrt{n}\frac{\bar{X}_n - \mu}{\sigma} \leq t\right) \to P(Z \leq t) \text{ for all } t \in \mathbb{R}, \text{ denoted by } \sqrt{n}\frac{\bar{X}_n - \mu}{\sigma} \xrightarrow{d} Z,$$

where Z has the standard normal distribution, $N(0,1)$. This was followed by a large number of generalizations upto finite-dimensional RV's. These are extremely useful in statistical inferences since typically F is unknown, but asymptotic probability statements could be made regardless of the parent distribution.

With more sophisticated inference procedures came the need for "Functional CLT's." For example, the sample DF, $F_n(x) = \frac{1}{n}\sum_{i=1}^n I(X_i \leq x)$, where $I(\cdot)$ is the Indicator (or characteristic) function is known to be an excellent estimator of F with $||F_n - F|| \xrightarrow{a.s.} 0$ and $\sqrt{n}[F_n(x) - F(x)] \xrightarrow{d} N(0, F(x)[1 - F(x)]) \; \forall \; x$. However, we need the asymptotic distribution of $\sqrt{n}||F_n - F||$ for the Kolmogorv-Smirnov test to deteremine if 2 DF's are the same or not. Although this was derived by difficult methods, a systematic procedure requires study of weak convergence of the entire function, $\sqrt{n}[F_n - F]$. Unfortunately, in studying convergences of measures in a metric space with the natural Borel topology we encounter problems with the measures not being Borel, or, equivalently, the random functions being nonmeasurable. Many *ad hoc* methods were used in the past. This will be an expository talk on a recent method developed by Hoffmann-Jørgensen that overcomes most of these problems.

Friday, March 11, 2005
3:00 PM in 372 Jabara Hall

Please come join us for refreshments before the lecture at 2:30 p.m. in room 353 Jabara Hall.